A) WEAK-FIELD (FLAT SPACETIME) LIMIT

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

I.E. $\alpha(r,t) = a(r,t) \equiv 1$, $\beta(r,t) \equiv 0$

• SCALAR FIELD E.O.M.

$$\Box \phi = 0 \quad \rightarrow \quad \phi_{tt} = \frac{1}{r^2}(r^2 \phi_r)_r$$

$$\boxed{(r\phi)_{tt} = (r\phi)_{rr}}$$

GENERAL SOLUTION (SCHEMATIC)

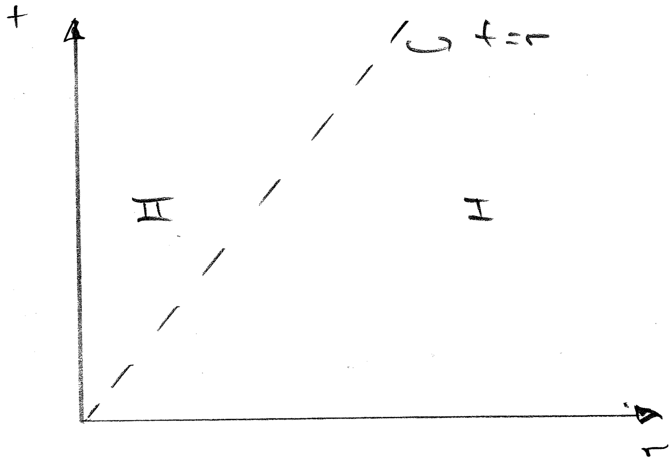
$$(r\phi)(r,t) = \underbrace{f(t+r)}_{\text{INGOING}} + \underbrace{g(t-r)}_{\text{OUTGOING}}$$

• SPECIFY INITIAL DATA IN TERMS OF INITIALLY INGOING PROFILE $f(r)$ AND INITIALLY OUTGOING " $g(r)$

$$(r\phi)(r,0) = f(r) + g(r)$$

$$(r\phi)_t(r,0) = f'(r) - g'(r)$$

COMPLETE SOLUTION



REGION I : $t \leq r$

$$(r\phi)(r,t) = f(t+r) + g(t-r)$$

REGION II : $t > r$ ONLY DATA WHICH WAS INCOMING AT $t=0$ CAN HAVE INFLUENCE HERE. SOL'N MUST STILL BE OF FORM

$$(r\phi)(r,t) = f(t+r) + h(t-r)$$

$$\rightarrow \quad \phi = \frac{f(t+r)}{r} + \frac{h(t-r)}{r}$$

NOW EXAMINE BEHAVIOUR NEAR $r=0$, ASSUME $\phi(0,t)$, f, h SMOOTH, THEN AS $r\to 0$

$$f(t+r) = f(t) + r f'(t) + \tfrac{1}{2} r^2 f''(t) + \ldots$$

$$h(t-r) = h(t) - r h'(t) + \tfrac{1}{2} r^2 h''(t) + \ldots$$

So, as $r \to 0$

$$\phi(t,r) = \frac{f(t) + h(t)}{r} + f'(t) - h'(t) + \frac{1}{2} r \left(f''(t) + h''(t) \right) + \ldots$$

Clearly, for the sol.^{n} to be regular at $r=0$ must have

$$h(t) = -f(t) \rightarrow h = -f \rightarrow h^{(n)} = -f^{(n)}$$

Then, as $r \to 0$

$$\phi(r,t) = 2f'(t) + \frac{1}{3} r^2 f'''(t) + \ldots$$

And we also recover

$$\phi_r(0,t) = 0$$

(regularity condition) as previously

Summary of complete (weak-field) sol^n

Given initial data
$$(r\phi)(r,0) = f(r) + g(r)$$
$$(r\phi)_t(r,0) = f'(r) - g'(r)$$

$$\boxed{(r\phi)(r,t) = \begin{cases} f(t+r) + g(t-r) & t \leq r \\ f(t+r) - f(t-r) & t > r \end{cases}}$$

B) STRONG-FIELD REGIME

- NO GENERAL CLOSED FORM SOL^N KNOWN (YET)

INITIAL DATA: SPECIFY $\Phi(r,0)$, $\Pi(r,0)$, TYPICALLY BY SPECIFYING $\phi(r,0) = f(r)$ AND ASSUME FLAT-SPACE PROPAGATION (AT $t=0$) AND INCOMING-ONLY DATA

- SOLVE CONSTRAINT ; SLICING EQN^S FOR $a(r,0)$, $\alpha(r,0)$

- EVOLVE DATA TO GENERATE S.T.

INTERPOLATING FAMILIES

- CONSIDER PARAMETERIZED FAMILIES OF INITIAL DATA

$$\Phi(r,0;p) \quad \Pi(r,0;p)$$

GENERATES SOL^N $S[p]$

EXAMPLE $\quad \phi(r,0) = \phi_0 r^3 \exp\left\{ - \left[\frac{r-r_0}{\Delta} \right]^q \right\}$

ANY OF ϕ_0, r_0, Δ, q CAN BE USED AS FAMILY PARAMETER

- HEURISTICALLY AS p DECREASES, GET WEAK FIELD LIMIT, AS P INCREASES, $S[p]$ IS STRONGER-FIELD, EVENTUALLY GET BLACK HOLE FORMATION (SOMETIMES ORIENTATION IS REVERSED, Δ FOR EXAMPLE)
- THESE FAMILIES GENERICALLY "INTERPOLATE" BETWEEN NO-BH AND BH SPACETIMES, BH FORMATION TURNS ON AT CRITICAL VALUE p^*

- INTERPOLATING FAMILIES SIp1 : SEE FIGURE of MASS ASPECT

- FOR GENERIC INTERP. FAMILIES (RECALL: $\Phi(r,0;p)$ $\Pi(r,0;p)$ ⇒ "INFINITE DIMENSIONAL" INITIAL-DATA SPACE) ALWAYS FIND THAT

$$\Pi \equiv \ln|p-p^*|$$

IS "NATURAL" PARAMETER FOR DESCRIBING PHENOMENOLOGY of SOLUTION SPACE AS $p \to p^*$

- NEAR-CRITICAL, STRONG-FIELD DYNAMICS (ALWAYS PLAYS OUT IN SOME NEIGHBORHOOD of $r=0$) CHARACTERIZED BY (ESSENTIALLY) UNIQUE SOL^N OF EMKG EQUATIONS

- CONVENIENT TO INTRODUCE NEW SCALAR FIELD VARIABLES

$$X(r,t) \equiv \sqrt{2\pi}\,\frac{r}{a}\,\Phi = \sqrt{2\pi}\,\frac{r}{a}\,\phi_r$$

$$Y(r,t) \equiv \sqrt{2\pi}\,\frac{r}{a}\,\Pi = \sqrt{2\pi}\,\frac{r}{\alpha}\,\phi_t$$

THEN

$$\frac{dm}{dr} = 4\pi r^2 \rho = X^2 + Y^2$$

$$R = -8\pi T = \frac{4}{r^2}\left(X^2 - Y^2\right)$$

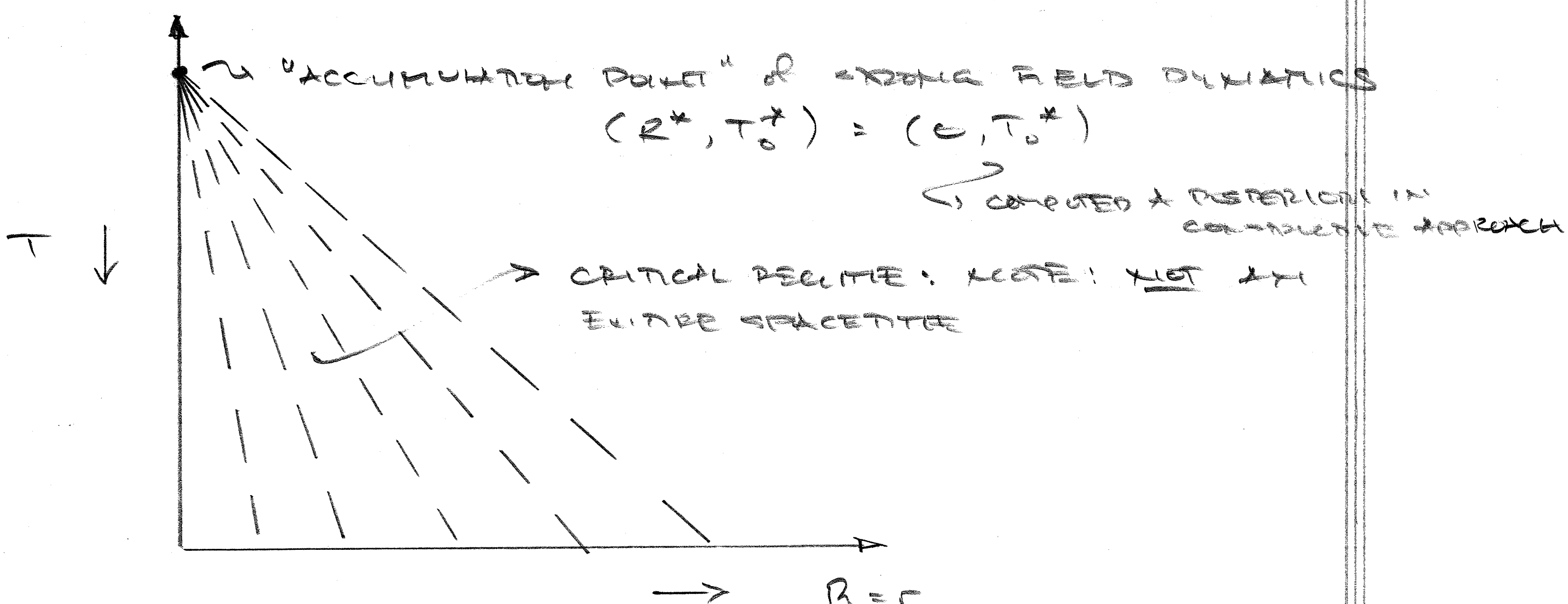
↳ 4-D RICCI SCALAR

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2$$

- NATURAL PARAMETERIZATION OF $t =$ CONST HYPERSURFACES FROM P.O.V. OF CRITICAL PHENOMENA → USE PROPER TIME OF CENTRAL OBSERVER (OBS. FIXED AT $r=0$)

$$T_0(t) = \int_0^t \alpha(0,\tilde{t})\, d\tilde{t}$$

- SCHEMATIC STRUCTURE OF CRITICAL SOLUTION



$$T = T_0^* - T_0$$

- DEFINE LOGARITHMIC COORDS

$$\tau \equiv |\ln T|$$

$$\rho \equiv |\ln R|$$

(ASSUME $T, R < 1$)

- CRITICAL SOLUTION: DENOTE SCHEM. AS

$$\underline{Z}^*(\rho, \tau)$$

$\underline{Z}^* = \{X^*, Y^*\}$, FOR EXAMPLE, SUFFICES TO COMPLETELY DESCRIBE SOL'N

- KEY PROPERTY: DISCRETE SELF-SIMILARITY (DSS) (aka "SCALE-PERIODICITY", "ECHOING")

$$\boxed{\underline{Z}^*(\rho, \tau) \simeq \underline{Z}^*(\rho \pm n\Delta, \tau \pm n\Delta)}$$

IN CONTEXT OF SOL'N GENERATED FROM (ASYMPTOTICALLY FLAT) GENERIC INTERPOLATING FAMILY, MEANS ~~THAT~~ EQUIV. HOLDS

a) IN LIMIT $p \to p^*$

b) IN LIMIT $\rho, \tau \to \infty$ $(R, T) \to (0,0)$

c) ONLY IN STRONG-FIELD CRITICAL REGIME (VALID ONLY WHERE VALID)

$\Delta = 3.44\ldots$ IS UNIVERSAL SCALING EXPONENT

DOES NOT DEPEND ON PARTICULAR INTERPOLATING FAMILY

- DYNAMICS REPEATS ON SCALES RELATED BY FACTOR $e^\Delta \sim 30$ (SEE FIGURE 3)

- PRECISELY CRITICAL SOL'N "ECHOES" INFINITE #

OF TIMES, SINGULAR AT $(R,T) = (0,0)$

- $X, Y, X^2+Y^2, X^2-Y^2, \ldots$ OSCILLATE BETWEEN FIXED, UNIVERSAL LIMITS → SOL^N REMAINS STRONG-FIELD DOWN TO ARBITRARILY SMALL SCALES

$$R = \frac{4}{r^2}\left(X^2 - Y^2\right) \to \infty$$

as $r \to 0$

APPARENT HORIZON NEVER FORMS IN PRECISELY-CRITICAL OR SUB-CRITICAL EVOLUTIONS; ALL SCALAR FIELD EVENTUALLY DISPERSES TO INFINITY → CRITICAL SING. IS "NAKED" (VISIBLE TO OBSERVERS AT ∞)

- CRITICAL SOL^N CLEARLY UNSTABLE (BY CONSTRUCTION), INF. PERTURBATIONS WILL RESULT IN EITHER COMPLETE DISPERSAL OR BH FORMATION

- ARBITRARILY SMALL BHs CAN BE PRODUCED VIA FINE-TUNING OF $p - p^*$ → BH TRANSITION IN EMKG MODEL IS TYPE II (ANALOGOUS TO 2ND-ORDER PHASE TRANS. IN STAT MECH SYSTEM, M_{BH} ~ ORDER PARAMETER)

- BLACK HOLE MASS SCALING: FIND

$$\boxed{M_{BH} \approx C_f\, |p - p^*|^{\gamma}} \qquad (*)$$

C_f = FAMILY-DEPENDENT CONSTANT

$\gamma = 0.37\ldots$ UNIVERSAL MASS SCALING EXPONENT

- PLAUSIBILITY / CONSISTENCY OF (*)

CONSIDER TWO FAMILIES

$$S[p] \quad , \quad S[q]$$

WHERE THE FAMILY PARAMETERS p, q ARE RELATED BY A SMOOTH TRANSFORMATION

$$q = q(p) \qquad \text{AND} \qquad q^* = q(p^*)$$

THEN (ASSUMING $q > q^*$ IS SUPERCRITICAL REGIME)

$$\lim_{q \to q^*} M_{BH} \sim c_q (q - q^*)^\gamma$$

$$= c_q \left(q(p) - q(p^*) \right)^\gamma$$

$$\sim c_q \left(\frac{dq}{dp}\Big|_{p=p^*} (p - p^*) \right)^\gamma$$

$$= c_q \frac{dq}{dp}(p^*)^\gamma (p - p^*)^\gamma$$

$$= c_p (p - p^*)^\gamma$$

- OFTEN GET ASKED, GIVEN A FAMILY $S[p]$ WITH CRIT. VALUE p^*, WHY CAN'T I REDEFINE

$$p \to \tilde{p} = |p - p^*|^\gamma$$

THEN

$$\eta_{BH} \sim \tilde{p}$$

ANSWER: YOU CAN, BUT FAMILY NOW REQUIRES A PRIORI INFINITE FINE-TUNING; MUST SPECIFY p^* TO ARBITRARY PRECISION; I.E. NEW INTERPOLATING FAMILY IS NOT GENERIC

* CONSTRUCTIVE APPROACH ILLUSTRATES THIS FACT NICELY; IF ONE TRIES TO REPARAM. A FAMILY AS ABOVE, THEN AS $\hat{p} \to \tilde{p}^*$, ANY PERTURBATION (INEVITABLE NUMERICALLY) WILL LEAD TO GENERIC SCALING BEHAVIOUR

* NOTE THAT THIS IS A BASIC OBSERVATION ABOUT "USUAL" CRITICAL PHENOMENA AS WELL, COULD TRY TO PLAY SAME GAME WITH SPIN SYSTEM

$$M \sim |T - T_c|^{\beta}$$

CAN'T (EXPERIMENTALLY) SPECIFY T_c A PRIORI TO ARBITRARY ACCURACY

PHY 387M CRITICAL COLLAPSE: SELF SIMILARITY ①

REFERENCE: GUNDLACH gr-qc/9712084 ; REFS THEREIN

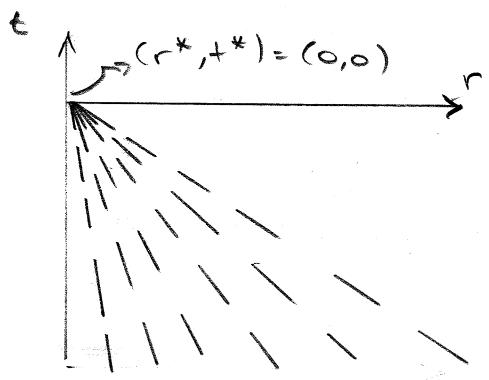
- SCHEMATIC PICTURE REVISITED

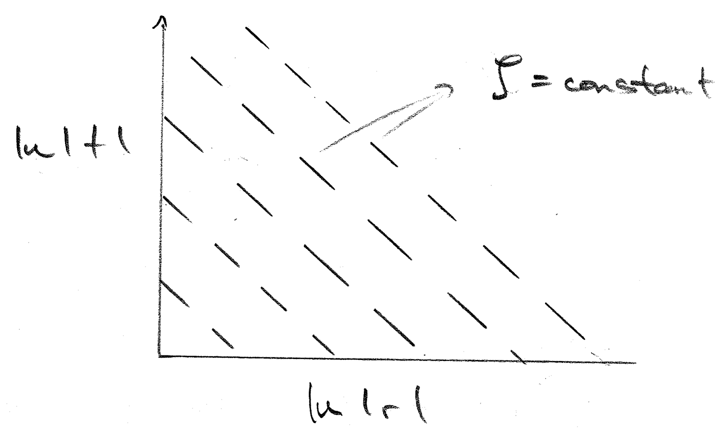
NOTE: IN TODAY'S DISCUSSION WILL (CONTINUE TO) ASSUME

a) SPHERICAL SYMMETRY

b) POLAR/AREAL COORDS WITH $\alpha(0,t) = 1$ $(t \to T_0)$

c) GUNDLACH'S SIGN CONVENTIONS AND VARIOUS NOTATIONS





NATURAL SIMILARITY VARIABLE (COORDINATE)

$$\boxed{\xi \equiv x \equiv -\frac{r}{t}}$$

NEED SECOND COORDINATE, COULD USE EITHER $\tau \sim \ln t$ or $\rho \sim \ln r$, FOLLOWING GUNDLACH WILL DEFINE

$$\boxed{\tau \equiv -\ln \frac{t}{L}}$$

→ IRRELEVANT, FIDUCIAL SPACETIME SCALE

- OBSERVE TWO TYPES of SELF-SIMILARITY IN TYPE II CRITICAL COLLAPSE

A) CONTINUOUS SELF-SIMILARITY (CSS)

$$\boxed{Z^*(x,\tau) = Z^*(x,\tau') = Z^*(x)}$$

OBSERVED IN COLLAPSE of

(i) PERFECT FLUID WITH E.O.S $P = k\rho$ (EVANS ; COLEMAN)

(ii) CERTAIN MULTI-SCALAR MODELS (LIEBLING ; CHOPTUIK)

B) DISCRETE SELF-SIMILARITY (DSS)

$$\boxed{Z^*(x,\tau+\Delta) = Z^*(x,\tau)}$$

"MODEL DEPENDENT ECHOING EXPONENT"

OBSERVED IN COLLAPSE of

(i) SINGLE MASSLESS SCALAR FIELD

(ii) AXISYMMETRIC GRAV WAVES (ABRAHAMS ; EVANS)

(iii) SU(2) YM FIELD (CHOPTUIK, CHMAJ ; BIZON)

- CSS CASE IS EASIER TO ANALYZE - KEY POINT IS THAT CSS ANSATZ REDUCES (IN SPH SYMM) PDE'S TO SET of ODEs + REGULARITY / ANALYTICITY $\Rightarrow$ "EIGENSOLUTIONS" $\equiv$ CRITICAL SOLUTIONS (OR AT LEAST CANDIDATES)

ASIDE: SELF-SIMILARITY IN GR

- WELL STUDIED TOPIC, PARTICULARLY IN SPH SYMM - SEE REFERENCES c) CARR ; COLEY b) CARR ; HENRIKSEN (TO BE PUBLISHED!) IN GUNDLACH

- HOWEVER, MOST OF THESE STUDIES DO NOT CONSIDER SS FROM "DYNAMICAL PERSPECTIVE" (EXTENT TO WHICH SOLN'S ACT AS "ATTRACTORS" etc.), SO RELEVANCE of SOLN'S TO BH FORMATION-SCENARIOS UNDERAPPRECIATED

COORDINATE-FREE DESCRIPTION of SS IN GR

(SEE WALD APP C, 443-444)

- CONFORMAL KILLING VECTORS

η^a IS A CONFORMAL KILLING VECTOR FIELD (CKV) OF A SPACETIME IF

$$\boxed{\mathcal{L}_\eta g_{ab} = \alpha g_{ab}}$$

FOR SOME FUNCTION α

$$\nabla_a \eta_b + \nabla_b \eta_a = \alpha g_{ab} \qquad (C.3.13)$$

- TAKE TRACE

$$2\nabla^c \eta_c = n\alpha \quad \Rightarrow \quad \alpha = \frac{2}{n}\nabla^c \eta_c$$

$$\boxed{\nabla_a \eta_b + \nabla_b \eta_a = \frac{2}{n} \nabla^c \eta_c \, g_{ab}} \qquad (C.3.14)$$

- HOMOTHETIC KILLING VECTORS

- SPECIAL CASE OF CKV: ξ^a IS A HOMOTHETIC KV IF

$$\boxed{\begin{gathered} \mathcal{L}_\xi g_{ab} = 2 g_{ab} \\ \nabla_a \xi_b + \nabla_b \xi_a = 2 g_{ab} \end{gathered}}$$

NOTE: THE CONSTANT "2" IS CONVENTIONAL AND CAN BE REPLACED WITH ANY ARB NON-ZERO VALUE VIA RESCALING OF ξ^a

THEN "SPACETIME HAS A CONTINUOUS SELF-SIMILARITY"

$\equiv$

"SPACETIME HAS A HOMOTHETIC KILLING VECTOR"

$\equiv$

"SPACETIME HAS A HOMOTHETY"

- CAN BE EXTENDED TO DSS CASE (GUNDLACH) BUT MUCH LESS STUDIED

EXERCISE: SHOW THAT IF ξ^a IS A HKV SO THAT $\mathcal{L}_\xi g_{ab} = \lambda g_{ab}$, λ CONSTANT THEN

$$\mathcal{L}_\xi R_{abcd} = \lambda R_{abcd}$$

$$\mathcal{L}_\xi R_{abc}{}^{d} = 0 \qquad \mathcal{L}_\xi G_{ab} = 0$$

OVERVIEW of ANALYSIS of TYPE II SOL^{N}S

- FOR GIVEN MODEL, WHICH INCLUDES SPEC. of
  - MATTER CONTENT
  - COUPLINGS
  - SYMMETRY

GENERICALLY FIND "ISOLATED" CRITICAL (THRESHOLD) SOLUTION, Z^*

- Z^* CAN BE CONSTRUCTED
  a) INDIRECTLY VIA DYNAMIC EVOLUTION of 1-PARAMETER INTERPOLATING FAMILIES
  b) DIRECTLY FROM ANSATZ REFLECTING PART. SELF-SIMILARITY (CSS or DSS) EXHIBITED BY SOL^{N}

- ~~CAN~~ THEN DO PERTURBATION THEORY USING Z^* AS BACKGROUND

- ASSUME Z^* IS CSS, THEN FOR A NEAR-CRITICAL SOL^{N} $Z(x,\tau)$, WE WILL HAVE, TO LINEAR ORDER

$$\boxed{\begin{aligned}\delta Z(x,\tau) &= Z(x,\tau) - Z^*(x,\tau)\\ &= \sum_{i=1}^{\infty} c_i\, e^{\lambda_i \tau} f_i(x)\end{aligned}}$$

$f_i(x)$: EIGENMODES

λ_i : CORRESPONDING EIGENVALUES

c_i : COEFFICIENTS

- CRUCIAL OBSERVATION CONCERNING TYPE II CRITICAL SOL^N WAS MADE BY KOIKE et al (PRL, 74, 5170 (1995)) "SHARPNESS" of PHASE TRANSITION SUGGESTED THAT ONLY ONE MODE $f_1(x)$ WAS UNSTABLE; I.E. HAD AN E.V. λ_1 WITH $\mathrm{Re}\,\lambda_1 > 0$ (EVANS ALSO HAD THE IDEA)

- THUS TYPE II CRITICAL SOL^N $\equiv$ MINIMALLY UNSTABLE SELF-SIMILAR SOL^N

- PROCEEDING ON THIS BASIS, WE THEN HAVE AS $p \to p^*$

$$Z(x,\tau;p) \simeq Z^*(x,\tau) + C_1(p)\, e^{\lambda_1 \tau} f_1(x)$$

$$C_1(p^*) = 0 \Rightarrow C_1(p) \simeq \left.\frac{dC_1}{dp}\right|_{p^*} (p-p^*) \equiv \frac{dC_1}{dp}(p-p^*)$$

$$\simeq Z^*(x,\tau) + \frac{dC_1}{dp}(p-p^*)\, e^{\lambda_1 \tau} f_1(x)$$

BLACK HOLE MASS SCALING (ASSUME THAT $\mathrm{Re}\,\lambda_1 = \lambda_1$)

- AS $p \to p^*$, $Z(x,\tau;p) \simeq Z^*(x,\tau)$ FOR MORE & MORE DECADES of SCALE, DEV. δZ GOVERNED BY $f_1(x)$ (ONE SIGN $\to$ DISPERSAL; OTHER SIGN $\to$ BH FORMATION)

- HEURISTICALLY, BH FORMATION WILL OCCUR WHEN $(p-p^*)\, e^{\lambda_1 \tau_p}$ REACHES SOME CRITICAL VALUE (INDEPENDENT of SCALE, τ_p) AND ALWAYS AT THAT SCALE τ_p

RECALL: $\tau \equiv -\ln \frac{t}{L} \Rightarrow t_p = L\, e^{-\tau_p} \simeq M_{BH}(p)$

THUS, CONDITION FOR BH FORMATION IS

$$(p-p^*)\, e^{\lambda_1 \tau_p} \simeq \text{const.} = K$$

$$\rightarrow \quad \ln(p-p^*) = -\lambda_1 \tau_p + \kappa \qquad \kappa = \ln K$$

$$= \lambda_1 \ln \frac{t_p}{L} + \kappa$$

$$\rightarrow \quad \ln r_{BH}(p) = \lambda_1^{-1} \ln(p-p^*) + \kappa'$$

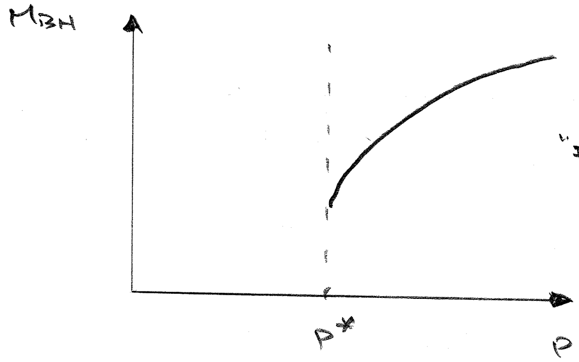
$$\boxed{r_{BH}(p) = c_f\, |p-p^*|^{1/\lambda_1}}$$

WHICH IS THE EMPIRICALLY DEDUCED SCALING LAW WITH

$$\boxed{\gamma = (\mathrm{Re}\,\lambda_1)^{-1}}$$

CRITICAL COLLAPSE: TYPE I SOL^{N}S

- RECALL: BY DEFN, WILL HAVE FOLLOWING BEHAVIOUR OF $M_{BH}(p)$ FOR TYPE I SOLNS:



"FIRST-ORDER PHASE TRANS."

- SO FAR, TYPE I SOL^{N}S ARE STATIC / QUASI-STATIC (PERIODIC) SOL^{N}S TO COUPLED EINSTEIN/MATTER EQN^{S} IN SPH. SYMM. RESTRICT DISCUSSION TO EXACTLY STATIC CASE

- EXAMPLE: SU(2) EINSTEIN-YANG-MILLS (EYM) (CHOPTUIK et al, PRL, 77, 424)

$$ds^2 = -\alpha^2(r,t)\,dt^2 + a^2(r,t)\,dr^2 + r^2 d\Omega^2$$

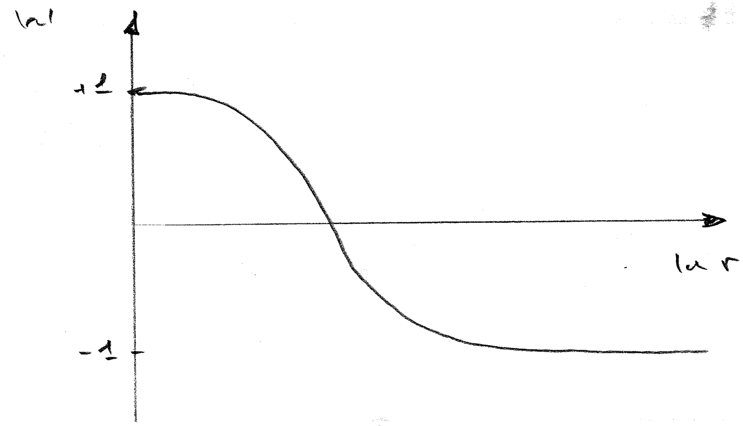
$$L_M = -\left(\frac{\nabla^\mu W \nabla_\mu W}{r^2} + \frac{1}{2}\frac{(1-W^2)^2}{r^4} \right)$$

$W(r,t) \equiv$ YANG-MILLS POTENTIAL

- STATIC SOL^{N}S: (BARTNIK & MCKINNON, PRL, 61, 141)

- ASSUME $\alpha = \alpha(r)$, $a = a(r)$, $W = W(r)$ - LOOK FOR SOL^{N}S OF EYM EQN'S WHICH ARE REGULAR AT $r = 0$ AND $r \to \infty$; IN PARTICULAR, MUST HAVE $W(0) = \pm 1$, $W(\infty) = \pm 1$

• FIND COUNTABLE INFINITY of STATIC SOL^NS $W_n(r)$, $n=1,2,3\ldots$
WHERE n COUNTS # of 0-CROSSINGS OF $W(r)$



• SUBSEQUENT STUDY SHOWED THAT ALL THE $W_n(r)$ STATIC SOL^NS ARE UNSTABLE WITH PRECISELY n UNSTABLE MODES IN PERT THEORY WITHIN PARTICULAR ANSATZ ("PURELY MAGNETIC")

• SUGGESTED TO BIZON THAT $W_1(r)$ MIGHT BE CRITICAL SOL^N IN SENSE WE HAVE BEEN DISCUSSING IT

• EXPECTATION BORNE OUT BY "USUAL" GAME INVOLVING 1-PARAMETER INTERPOLATING FAMILIES GENERATED FROM INITIAL DATA $W(r,0;p)$

<u>NOTE</u>: $W(r,0;p)$ DOES NOT NECESSARILY HAVE TO BE A "KINK" IN ORDER FOR $W(r,0;p^*) \simeq W_1(r)$

• <u>PHENOMENOLOGY</u>

$$\text{AS } p \to p^*, \quad W(r,t) \to W_1(r)$$

• <u>SCALING LAW</u>: "LIFETIME", τ, OF CONFIGURATION

$$\boxed{\tau \sim -\sigma \ln|p-p^*|}$$

σ: UNIVERSAL EXPONENT

- PERTURBATION THEORY : THIS TIME, IT WAS ALREADY KNOWN THAT (AGAIN, WITHIN PART. SELF-CONSISTENT ANSATZ), $u_1(r)$ HAD ONLY ONE UNSTABLE MODE

$$Z(r,t;p) \simeq Z^*(r,t) + C_1(p)\, e^{\lambda_1 t} f_1(r)$$

$$\simeq Z^*(r,t) + \left.\frac{dC_1}{dp}\right|_{p^*} \underbrace{(p-p^*)\, e^{\lambda_1 t}}\, f_1(r)$$

- USING THE SAME ARG. AS FOR TYPE II CASE, BH FORMATION (or DISPERSAL) SIGNALLED BY

$$(p-p^*)\, e^{\lambda_1 t_p} = K$$

$$e^{\ln(p-p^*)}\, e^{\lambda_1 t_p} = K$$

$$\Rightarrow \quad \boxed{t_p = -\frac{1}{\lambda_1} \ln(p-p^*)} \qquad (t_p = \tau)$$

So $\quad \boxed{\sigma = \frac{1}{\lambda_1}}$

- NOTE : MARGINALLY SUPER-CRITICAL EVOLUTIONS

$$M_{BH} \simeq M_1 \equiv \text{TOTAL MASS (ADM MASS) OF STATIC } n=1 \text{ CONFIGURATION}$$

• MEASUREMENT of LIFETIME: AGAIN, CRUCIAL OBSERVATION IS THAT FOR $p_1 \simeq p^*$, $p_2 \simeq p^*$ $|p_1 - p^*| > |p_2 - p^*|$, LATE-TIME p_2 EVOLUTION WILL QUITE PRECISELY MATCH LATE-TIME p_1 EVOLUTION (DEPARTURE FROM CRITICALITY IS "UNIVERSAL" - I.E. DESCRIBED BY $e^{\lambda_1 t} f_1(r)$) PROVIDED WE SHIFT p_2 EVOLUTION IN TIME TO MATCH RESPECTIVE PEEL-OFF TIMES. THUS

$$\sigma \simeq \frac{\delta t_p}{\delta \ln(p-p^*)}$$

CAN BE COMPUTED BY LOOKING ~~AT~~ SOME FEATURE IN THE NON-CRITICAL (I.E. LATE-TIME) REGIME. FOR EXAMPLE, DEFINE

$$T_r(\ln(p-p^*)) \quad \text{FOR SUB-CRITICAL EVOLUTIONS}$$

TO BE THE CENTRAL PROPER TIME AT WHICH THE 0-CROSSING OF W ARRIVES AT RADIUS r AS IT PROPAGATES OUTWARD (I.E. $W(r, T_r(\ln(p-p^*))) = 0$), THEN AS $p \to p^*$

$$-\frac{dT_r}{d\ln(p-p^*)} \to \sigma$$

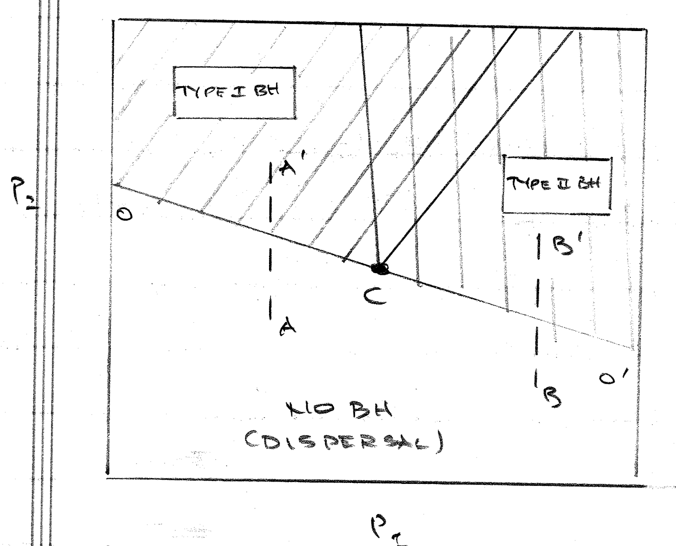
TYPICAL INTERP. FAMILY: $\sigma \sim 0.55(2)$

PERTURBATION THEORY: $\sigma = 0.5519\ldots$

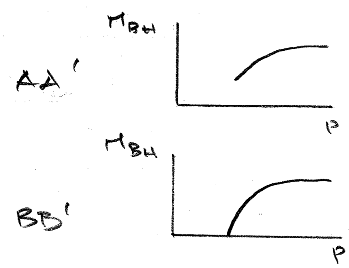
2-PARAMETER FAMILIES AND TYPE I ↔ TYPE II TRANS.

- SUITABLE 2-PARAM. FAMILIES OF INITIAL DATA $W(r, 0; p_1, p_2)$ CAN GENERATE BOTH TYPE I AND TYPE II BEHAVIOUR

"PHASE DIAGRAM"



OO': CRITICAL LINE



C: CO-EXISTENCE POINT (NAKED SINGULARITY INSIDE n=1 B-MK SOLUTION)

- EXAMPLE: EINSTEIN-MASSIVE-KLEIN-GORDON COLLAPSE (BRADY et al, PRD, 56, 6057)

$$\Box \phi(r,t) = m^2 \phi(r,t)$$

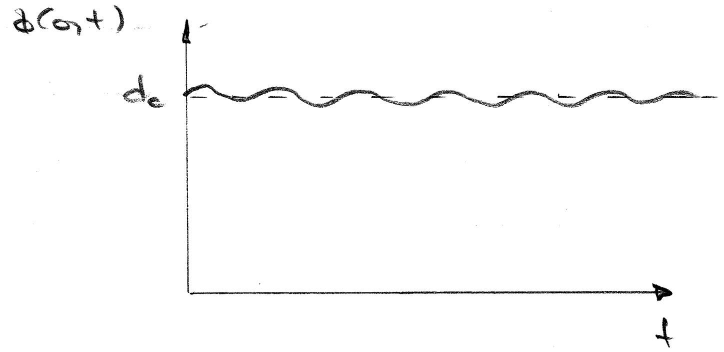
$$L_M = -\left(\nabla^\mu \phi \nabla_\mu \phi + \tfrac{1}{2} m^2 \phi^2\right)$$

- SEIDEL & SUEN (PRL, 66, 1659) HAD PREVIOUSLY DISCOVERED PERIODIC (QUASI-STATIC) SOLNS TO EKG EQUATIONS WHICH THEY DUBBED "OSCILLON STARS"

$$g(r,t) \sim \sum_{n=0}^{\infty} e^{in\omega t} f_n(r)$$

↳ "EIGENFREQUENCY"

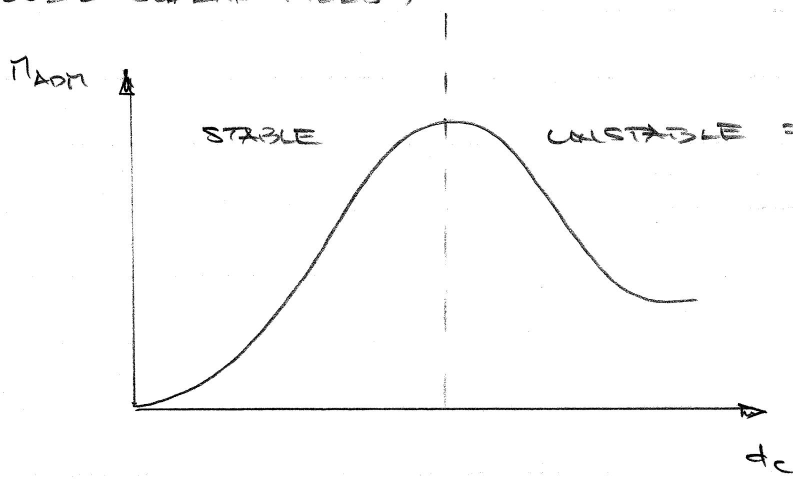
WITH $|f_0(r)| \gg |f_n(r)|$ $\quad n > 0$



INCORRECT!

ϕ HAS NO "DC" COMPONENT

- THESE OSCILLATONS FORM A 1-PARAMETER FAMILY, LABELED, FOR EXAMPLE BY $\phi(0,t) \equiv \phi_c$ (MEAN CENTRAL VALUE of SCALAR FIELD)



UNSTABLE ⇒ TYPE I CRITICAL SOL'NS

- THIS TIME THERE IS A CONTINUOUS FAMILY of TYPE I CRIT. SOL'NS (NOT JUST 1 AS IN EYM CASE)

- TYPE II BEHAVIOUR → TYPE II CRIT. SOL'N PRECISELY THE SAME AS Z^* FOR EMKG → FIELD NATURALLY GETS DRIVEN TO MASSLESS ≡ SCALE FREE ≡ KINETIC-ENERGY-DOMINATED STATE AS $p \to p^*$

ϕ^* BOUNDED $\Rightarrow V(\phi^*) = \frac{1}{2} m^2 \phi^{*2}$ BOUNDED

$\nabla^\mu \phi \nabla_\mu \phi$ UNBOUNDED ⇒ K.E. DOMINATES

⇒ TYPE OF UV ASYMPTOTIC FREEDOM