PHYS 170 Section 101
October Midterm Review
October 9, 2018

WARNING / DISCLAIMER

The instructor does not guarantee that the following review is complete. In particular, concepts and/or equations pertinent to the midterm *may* have been omitted below.

CHAPTER 1
GENERAL PRINCIPLES

NUMERICAL CALCULATIONS

- Rounding numbers
  - Number that ends in 5:

    "Round to even digit rule": 2.345 -> 2.34 (round down), but 2.355 -> 2.36 (round up)

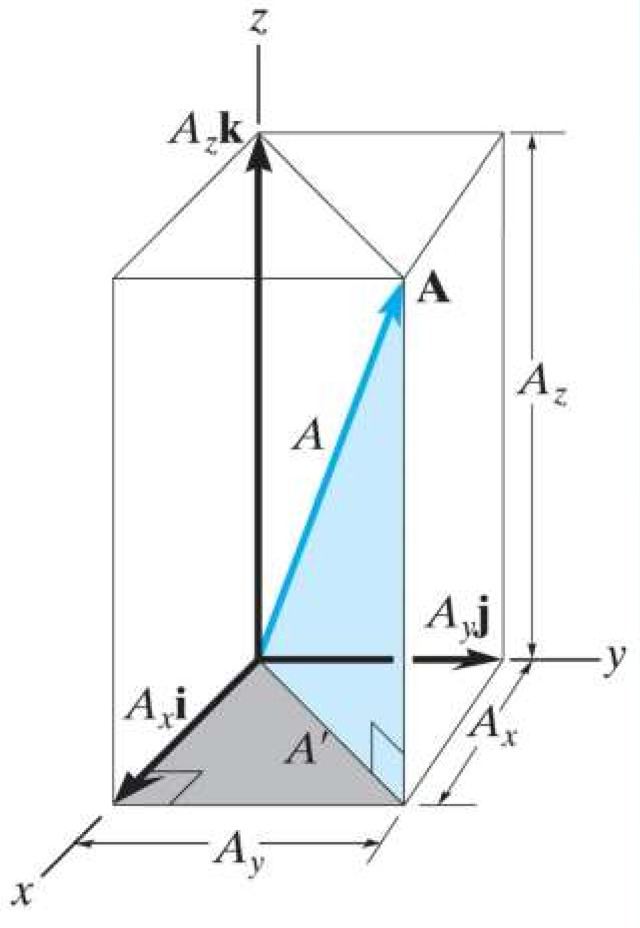
- Significant figures (sig. figs.)
  - Default for final answers: **Exactly 3 sig. figs.** (generally carry 4 or more for intermediate values)

- Keep numerical values of answers between .1 and 1000, use appropriate SI prefix when numbers are larger or smaller

  Example: 2450 N -> 2.45 kN

CHAPTER 2
FORCE VECTORS

CARTESIAN VECTOR REPRESENTATION & MAGNITUDE OF A CARTESIAN VECTOR

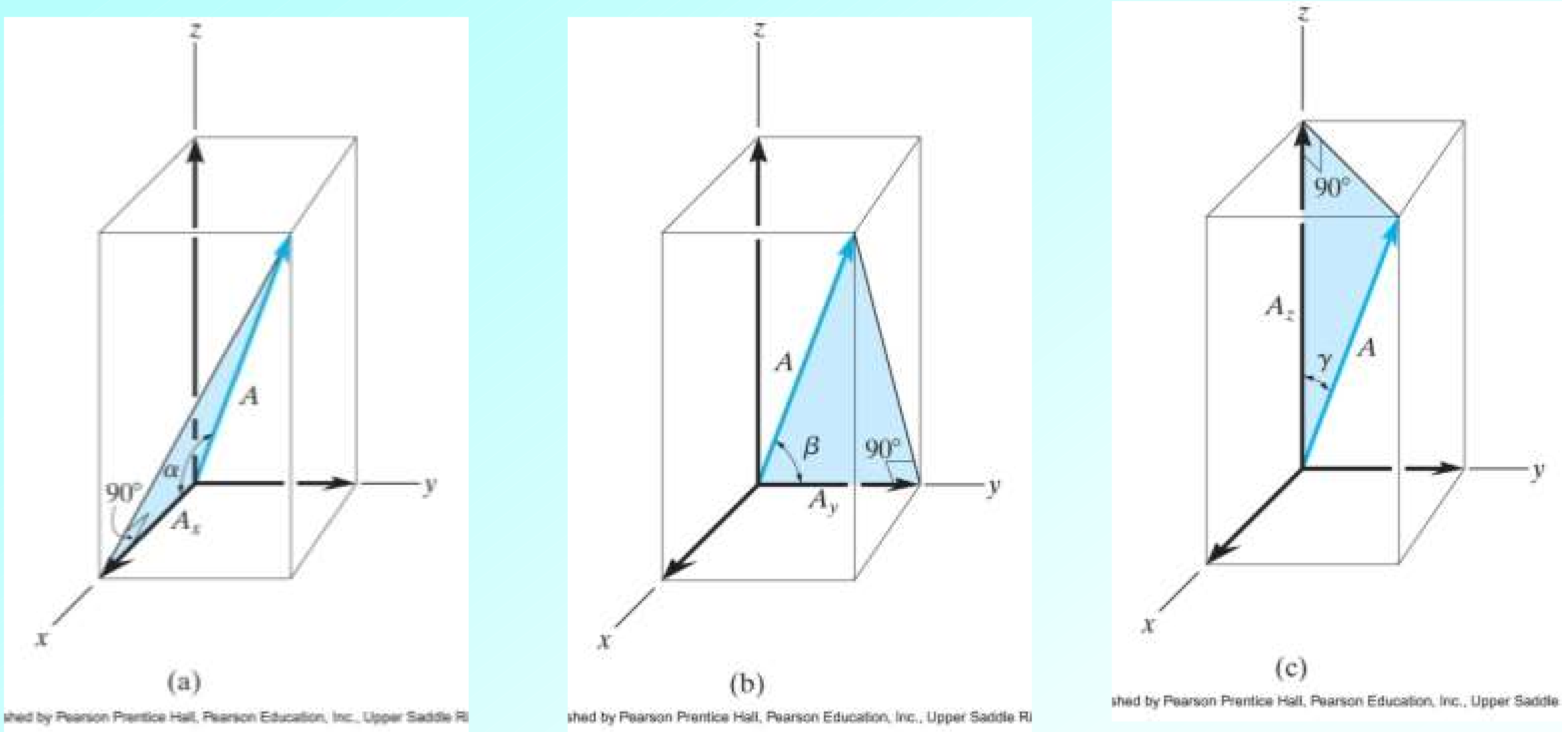


02_025

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

DIRECTION COSINES



$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Unit vector in direction of **A**

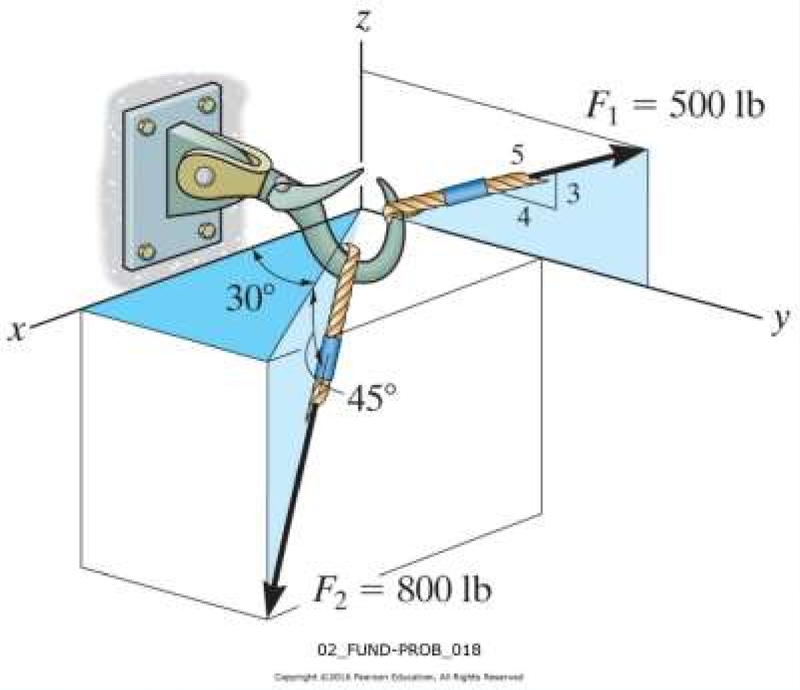
$$\mathbf{u}_A = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

$$\mathbf{A} = A\cos\alpha\mathbf{i} + A\cos\beta\mathbf{j} + A\cos\gamma\mathbf{k}$$

RESULTANT FORCE

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\,\mathbf{i} + \Sigma F_y\,\mathbf{j} + \Sigma F_z\,\mathbf{k}$$

Know how to interpret this type of figure



Determine the resultant force acting on the block

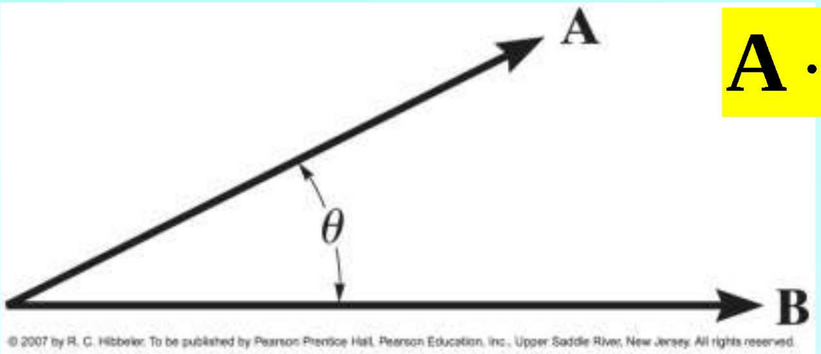
POSITION VECTORS

$$\mathbf{r} = \mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A$$
$$= (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

FORCE VECTOR DIRECTED ALONG A LINE

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$
$$= F\left(\frac{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

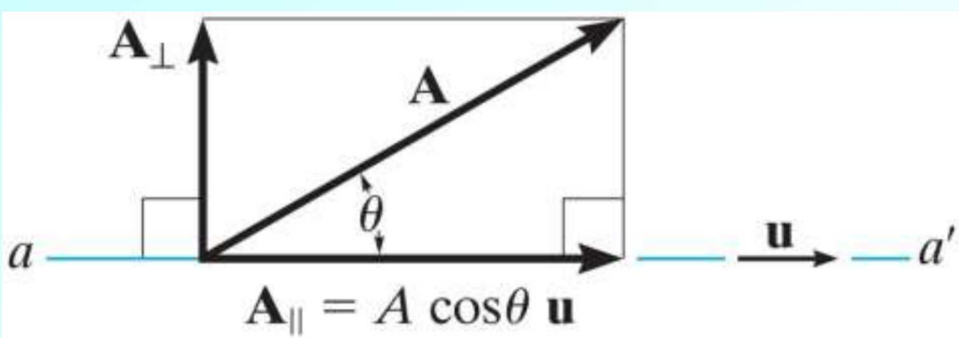
DOT PRODUCT



$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta \quad (0^\circ \leq \theta \leq 180^\circ)$$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$



$$A_{||} = \mathbf{A}\cdot\mathbf{u}$$

CHAPTER 3
EQUILIBRIUM OF A PARTICLE

EQUILIBRIUM OF A PARTICLE

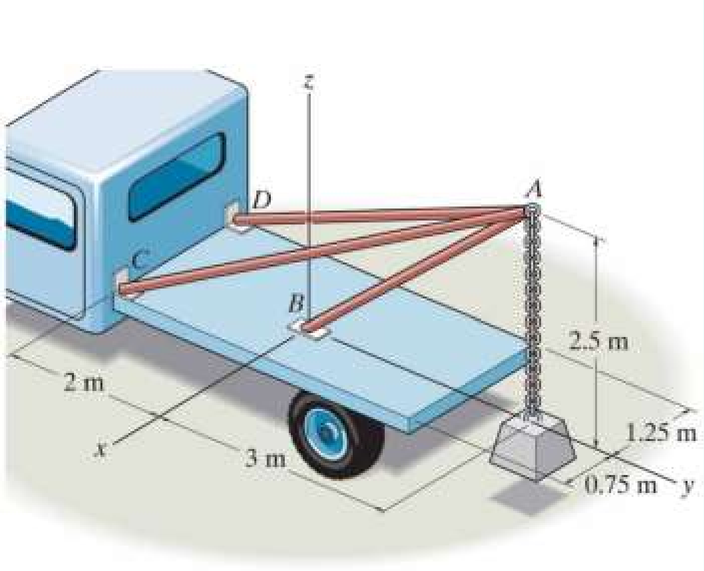
$$\Sigma \mathbf{F} = \mathbf{0}$$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

3D FREE BODY DIAGRAMS



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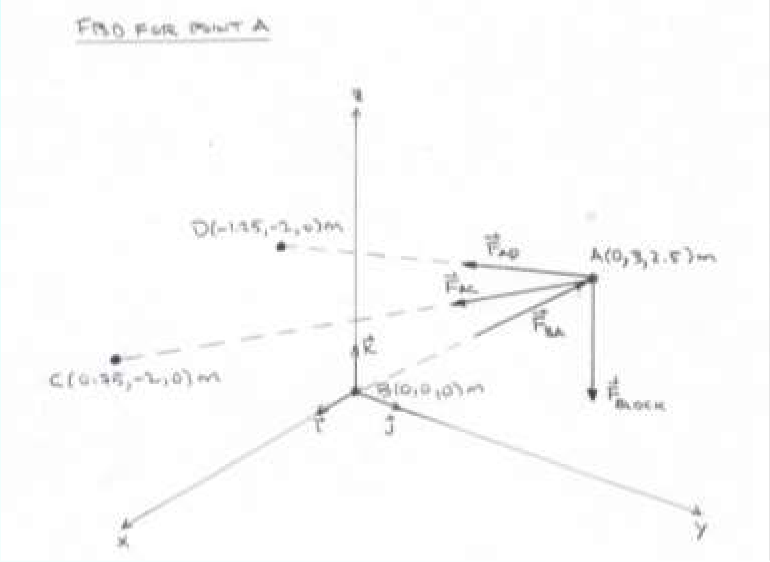


(No springs on midterm)

Solving equations using the reduced row echelon form method

- Equations for X, Y and Z :

$$0.75Y - 1.25Z = 0 \qquad (1)$$

$$3X - 5Y - 5Z = 0 \qquad (2)$$

$$2.5X - 2.5Y - 2.5Z = 4905 \qquad (3)$$

- Solve (1) to (3) using the reduced row echelon form matrix program **rref(**[M]**)** on a TI graphing calculator, where [M] is the 3 x 4 matrix

$$\begin{array}{cc} & \begin{array}{cccc} X & Y & Z & \text{rhs} \end{array} \\ [M] = & \begin{bmatrix} 0 & 0.75 & -1.25 & 0 \\ 3 & -5 & -5 & 0 \\ 2.5 & -2.5 & -2.5 & 4905 \end{bmatrix} \end{array}$$

Each **row** of the matrix corresponds to one equation; row elements must be entered consistently (i.e. X coefficients in first column, Y coefficients in second etc.); right hand side of the equation is always the last element of the row.

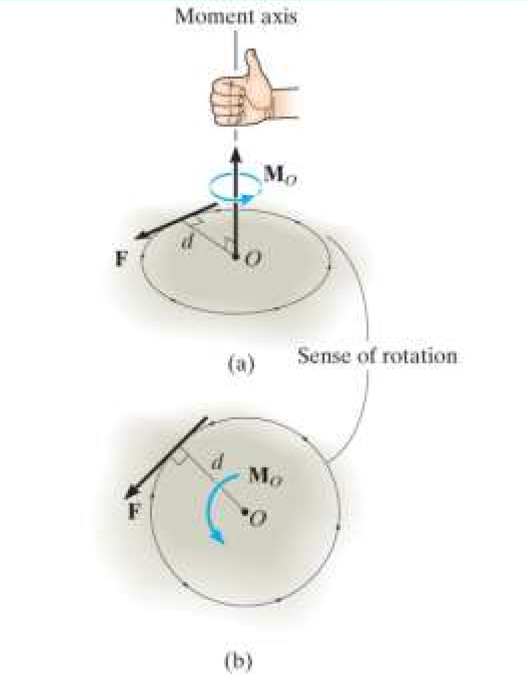
This yields

$$\begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4905 \\ 0 & 1 & 0 & 1839 \\ 0 & 0 & 1 & 1104 \end{bmatrix}$$

Last **column** of matrix returned by **rref**[M] contains solution, in same order, top to bottom, as unknowns in any row.

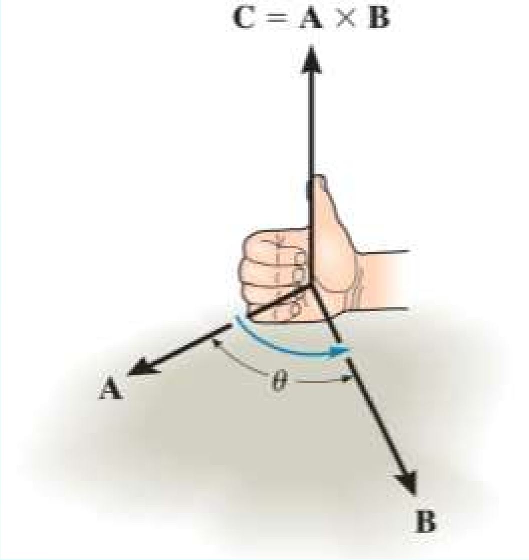
CHAPTER 4
FORCE SYSTEM RESULTANTS

MOMENT OF A FORCE



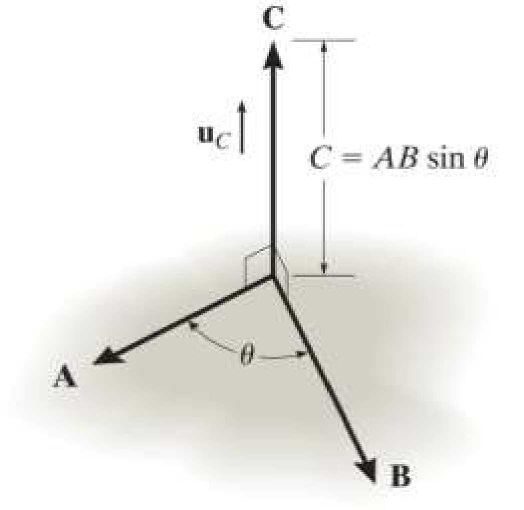
$$M_O = Fd$$

CROSS PRODUCT



$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

CROSS PRODUCT

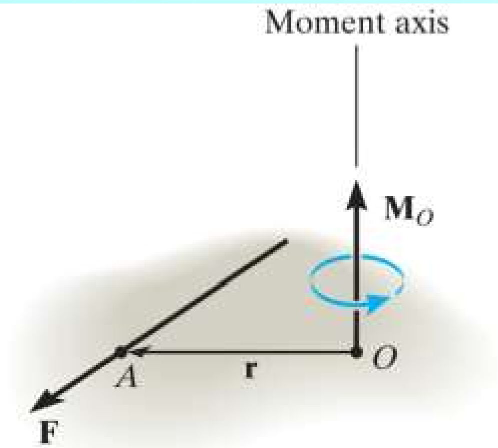


$$C = AB\sin\theta \qquad (0^\circ \le \theta \le 180^\circ)$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

MOMENT OF A FORCE

- Moment of force **F** about moment axis passing through *O* and perpendicular to the plane containing *O* and **F** is given by



(a)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

NOTE: **r** is a vector **from** *O* to **any** point on the line of action of **F**. Often pays to think about this before proceeding to compute moments!

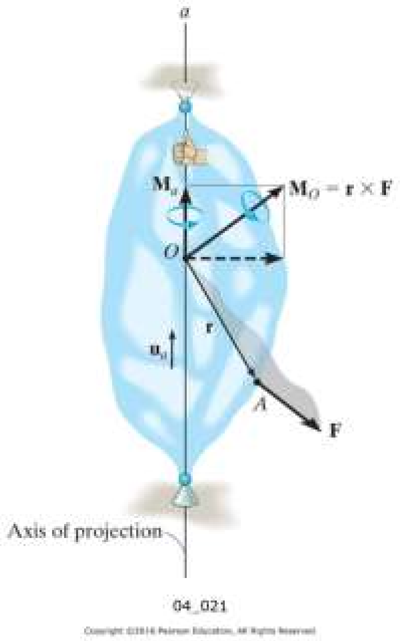
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (r_y F_z - r_z F_y)\,\mathbf{i} - (r_x F_z - r_z F_x)\,\mathbf{j} + (r_x F_y - r_y F_x)\,\mathbf{k}$$

RESULTANT MOMENT OF SYSTEM OF FORCES

$$\left(\mathbf{M_R}\right)_O = \Sigma(\mathbf{r}\times\mathbf{F})$$
$$= \mathbf{r}_{OA}\times\mathbf{F}_A + \mathbf{r}_{OB}\times\mathbf{F}_B + \mathbf{r}_{OC}\times\mathbf{F}_C + \ldots$$

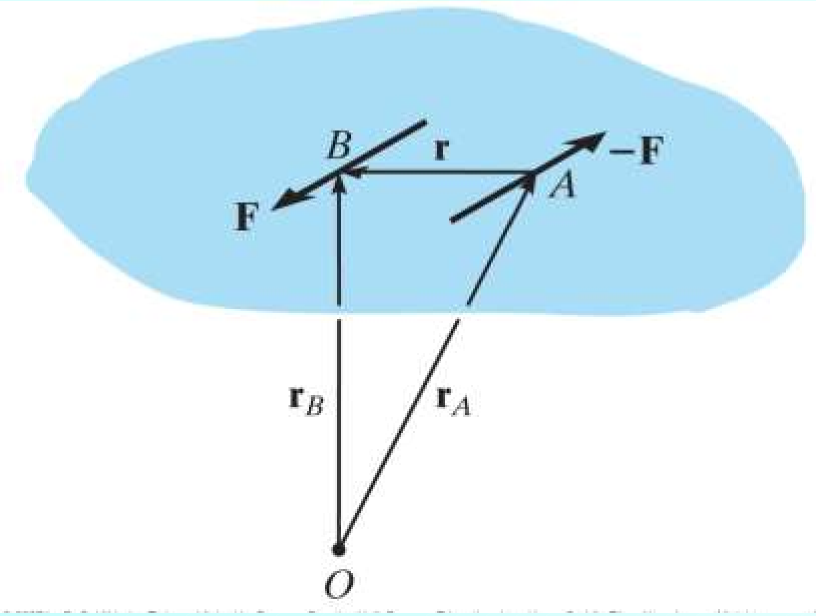
MOMENT OF A FORCE ABOUT A SPECIFIED AXIS



$$M_a = \mathbf{M}_O\cdot\mathbf{u}_a = \mathbf{u}_a\cdot\mathbf{M}_O = \mathbf{u}_a\cdot(\mathbf{r}\times\mathbf{F})$$

$$M_a = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A COUPLE



$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

RESULTANT COUPLE MOMENT

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$

RESULTANTS OF A FORCE AND COUPLE SYSTEM

Equivalent resultant force: $\mathbf{F}_R = \sum \mathbf{F}$

Equivalent resultant couple moment: $(\mathbf{M}_R)_O = \sum \mathbf{M} + \sum \mathbf{r} \times \mathbf{F}$

WRENCH

A force and couple moment system in which the resultant force and resultant couple moment are parallel (or anti-parallel) to one another.

Two vectors $\vec{V}$ and $\vec{W}$ are parallel (or anti-parallel) if $\vec{V} = c\vec{W}$ for some constant c.

Example:

$\vec{V} = 2\vec{i} - 6\vec{j} + 12\vec{k}$

$\vec{W} = 3\vec{i} - 9\vec{j} + 18\vec{k}$

$\vec{V} = \frac{2}{3}\vec{W}$ so the vectors are parallel ($c = \frac{2}{3}$, when $c < 0$ vectors are anti-parallel)

MISCELLANEOUS

- Solving systems of equations
  - Don't necessarily need to use calculator if you think "by hand" approach will be faster

- Setting up systems of equations
  - Don't necessarily need to use "trick" for simplifying equations when direction of forces are known but magnitudes aren't (introduction of X, Y, Z, etc.)
    If you prefer the straightforward, direct approach, use it.