PHYS 170 Section 101

Lecture 32

November 26, 2018

Nov 26—Announcements

- Final exam review sessions tomorrow in tutorial slots

- Modification of overall grading scheme:

  If your mark on the final exam is greater than your course mark computed using the original grading scheme, then your final grade will be equal to your mark on the final exam

- This will be the only modification, so please don’t request others

Nov 26—Announcements

- If you have not yet done so, please take the next 15 minutes to complete the evaluation of this course at

  Canvas -> Phys170.101 -> Course Evaluation

Lecture Outline/Learning Goals

- Second worked example of oblique impact
- Supplementary material for impact

Oblique Impact—Summary

Given the masses m_A , m_B , the coefficient of restitution, e, and the initial velocity components v_{A1n} , v_{A1t} , v_{B1n} and v_{B1t} , where n is the (normal) axis along the line of action, and t is the (transverse) axis perpendicular to n, the final velocity components are given by

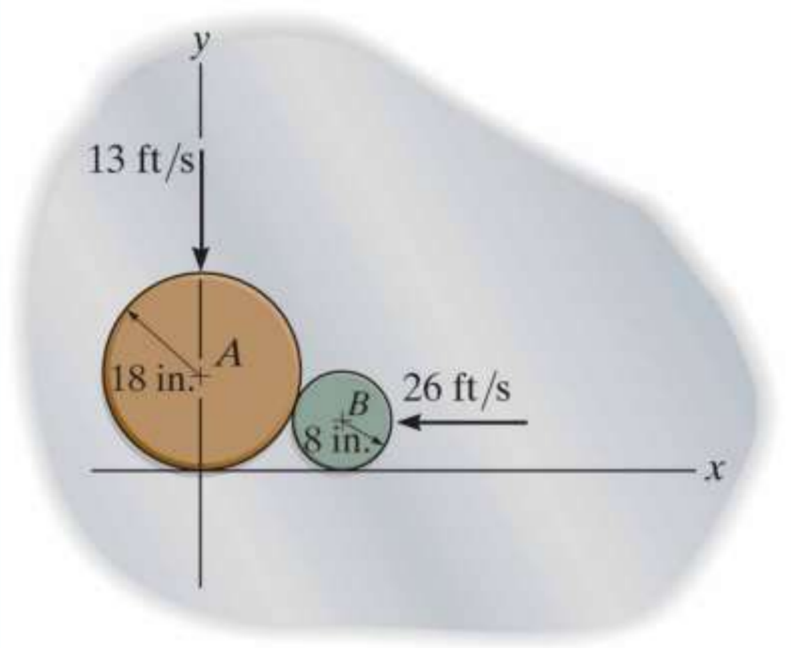
$$v_{A2n} = \frac{(m_A - em_B)v_{A1n} + m_B(1+e)v_{B1n}}{m_A + m_B}$$

$$v_{A2t} = v_{A1t}$$

$$v_{B2n} = \frac{(m_B - em_A)v_{B1n} + m_A(1+e)v_{A1n}}{m_B + m_A}$$

$$v_{B2t} = v_{B1t}$$

Problem 15-87 (Page 261, 12th edition)



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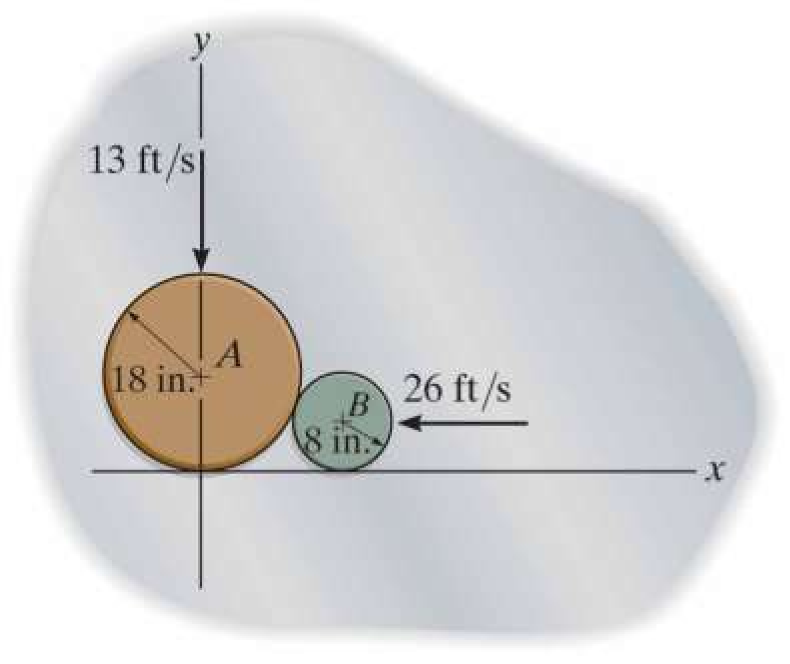
Discs *A* and *B* weigh 8 lb and 2 lb, respectively. They are sliding on the smooth horizontal plane with velocities as shown. The coefficient of restitution for the collision is 0.5

(1) Determine the velocities of *A* and *B* just after impact

(2) Express the velocities as Cartesian vectors in terms of speeds and angles with respect to the *x*-axis



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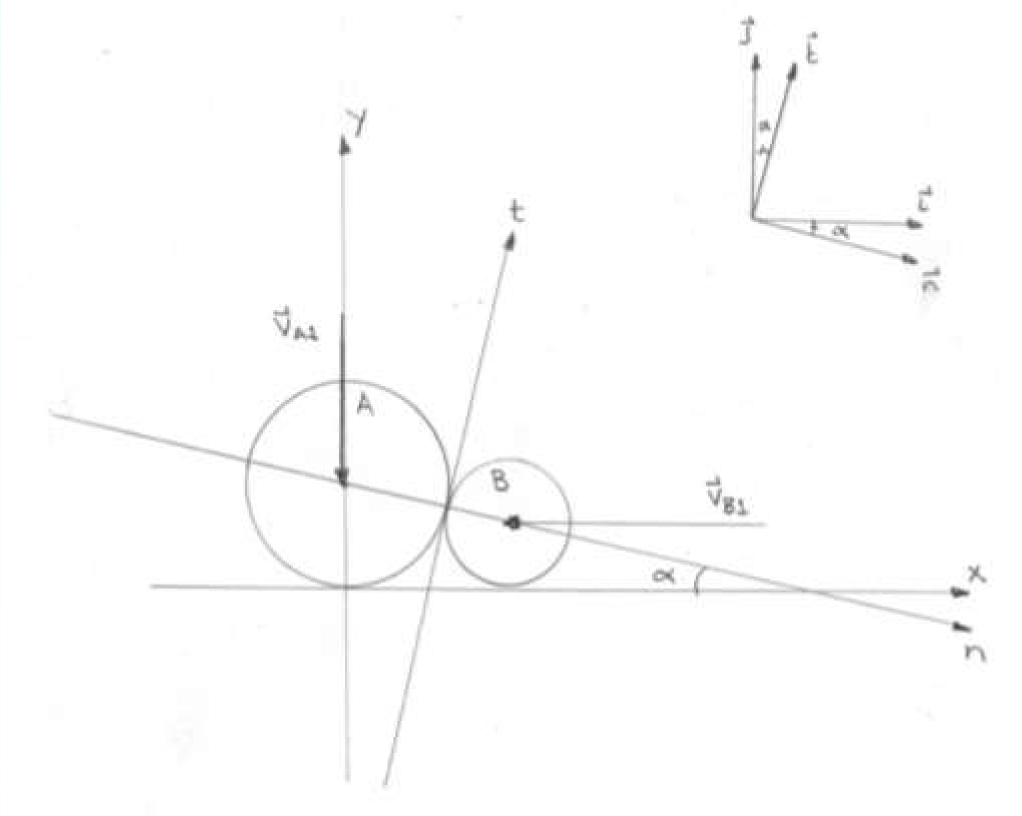
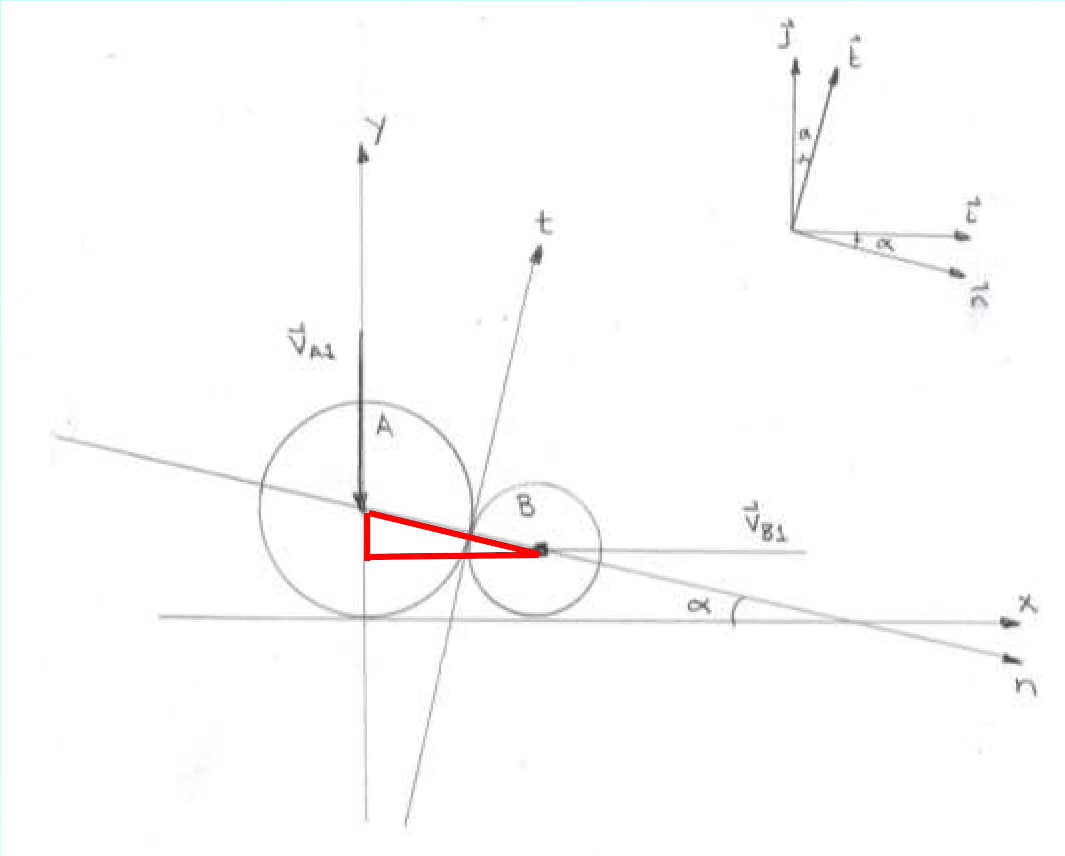




Solution strategy

Write down initial velocities in *nt* coordinates

Compute final velocity components using equations derived above

Express final velocities as Cartesian vectors in terms of speeds and angles with respect to the *x*-axis

Note: The right triangle outlined in red in the figure is a 5-12-13 triangle with least angle α. Thus we have $\sin\alpha = 5/13$ and $\cos\alpha = 12/13$. (The actual sides of the triangle are 10, 24 and 26 in.)

Data

$W_A = 8 \text{ lb}$ $\quad W_B = 2 \text{ lb}$ $\quad e = 0.5$

$\vec{v}_{A1} = -13\vec{j} \text{ ft/s}$ $\quad \vec{v}_{B1} = -26\vec{i} \text{ ft/s}$

$\alpha = \sin^{-1}(5/13) = 22.62^{\circ}$

$\sin\alpha = 5/13$ $\quad \cos\alpha = 12/13$

$\vec{i} = \cos\alpha\vec{n} + \sin\alpha\vec{t}$ $\quad \vec{j} = -\sin\alpha\vec{n} + \cos\alpha\vec{t}$

Initial velocities (suppressing units)

$$\vec{v}_{A1} = -13\vec{j} = 5\vec{n} - 12\vec{t}$$

$$\vec{v}_{B1} = -26\vec{i} = -24\vec{n} - 10\vec{t}$$

Final velocity components

$$v_{A2t} = v_{A1t} = -12$$
$$v_{B2t} = v_{B1t} = -10$$

Note: The equations for v_{A2n} and v_{B2n} contain a factor of mass in each term (both numerator and denominator). We can therefore use weights equally well as masses in the calculations (the factors of g cancel):

$$v_{A2n} = \frac{(m_A - em_B)v_{A1n} + m_B(1+e)v_{B1n}}{m_A + m_B} = \frac{[8-(0.5)2]5 + 2(1+0.5)(-24)}{8+2}$$
$$= -3.7$$
$$v_{B2n} = \frac{(m_B - em_A)v_{B1n} + m_A(1+e)v_{A1n}}{m_B + m_A} = \frac{[2-0.5(8)](-24) + 8(1+0.5)5}{2+8}$$

$$= 10.8$$

Final velocities (in *nt* coordinates)

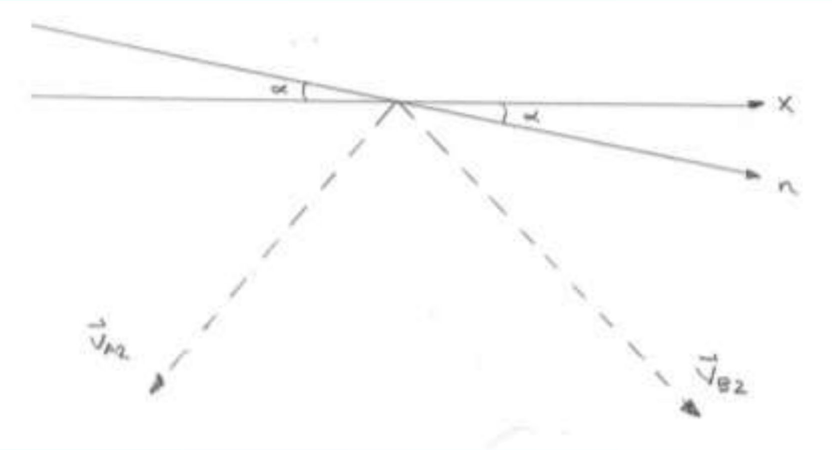
$$\vec{v}_{A2} = -3.70\vec{n} - 12.0\vec{t}$$

$$= 12.6(-\cos 72.9^\circ \vec{n} - \sin 72.9^\circ \vec{t}) \text{ ft/s}$$

$$\vec{v}_{B2} = 10.8\vec{n} - 10.0\vec{t}$$

$$= 14.7(\cos 42.8^\circ \vec{n} - \sin 42.8^\circ \vec{t}) \text{ ft/s}$$

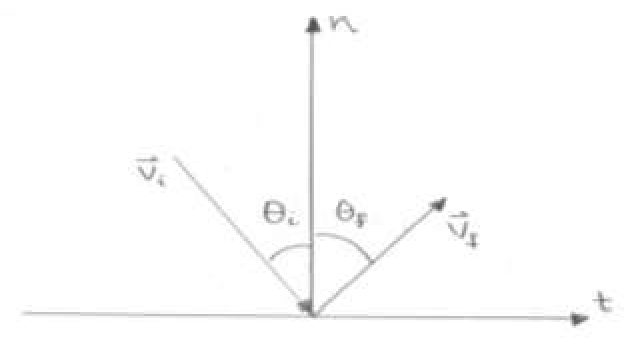
Final velocities as Cartesian vectors



$$\vec{v}_{A2} = 12.6(-\cos 50.2^\circ \vec{i} - \sin 50.2^\circ \vec{j}) \text{ ft/s}$$

$$v_{B2} = 14.7(\cos 65.4^\circ \vec{i} - \sin 65.4^\circ \vec{j}) \text{ ft/s}$$

Impact—Supplementary Material
(for understanding solutions of posted Additional Problems for this section in particular)



- Consider collision of an object with another object of much larger mass (momentum), such as a ball bouncing off the floor

- Then can set pre- and post-collision velocities of massive object to 0

- Referring to the diagram, we then have from the equation relating pre- and post-collision velocities components in the *n* direction to the coefficient of restitution (*i* = initial, *f* = final)

$$e = -\frac{v_f \cos\theta_f}{v_i \cos\theta_i}$$

and from conservation of momentum in the *t* direction

$$v_i \sin\theta_i = v_f \sin\theta_f$$

Manipulating these two equations yields

$$v_i^2 \sin^2 \theta_i = v_f^2 \sin^2 \theta_f$$

$$v_i^2 e^2 \cos^2 \theta_i = v_f^2 \cos^2 \theta_f$$

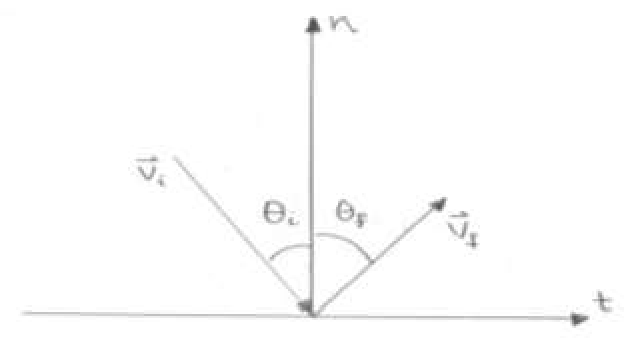
Adding these last two results gives

$$v_f^2 = v_i^2 (\sin^2 \theta_i + e^2 \cos^2 \theta_i)$$

so we have a formula for the final speed

$$v_f = v_i \sqrt{\sin^2 \theta_i + e^2 \cos^2 \theta_i}$$

Consider the diagram again, and write the equation relating the pre- and post-collision velocity components in the n direction to the coefficient of restitution as



$$e = \frac{v_{nf}}{v_{ni}}$$

where v_{nf} and v_{ni} are taken to be positive. From the diagram, we have

$$\tan\theta_i = \frac{v_{ti}}{v_{ni}}$$

$$\tan\theta_f = \frac{v_{tf}}{v_{nf}}$$

From conservation of momentum in the t direction we have

$$v_{tf} = v_{ti}$$

Putting these results together, we find

$$\frac{1}{e}\tan\theta_i = \frac{v_{ni}}{v_{nf}}\frac{v_{ti}}{v_{ni}} = \frac{v_{ti}}{v_{nf}} = \frac{v_{tf}}{v_{nf}} = \tan\theta_f$$

So we have an equation for the rebound angle

$$\tan\theta_f = \frac{1}{e}\tan\theta_i$$

$$\theta_f = \tan^{-1}\left(\frac{1}{e}\tan\theta_i\right)$$