PHYS 170 Section 101

Lecture 31

November 23, 2018

Nov 23—Announcements

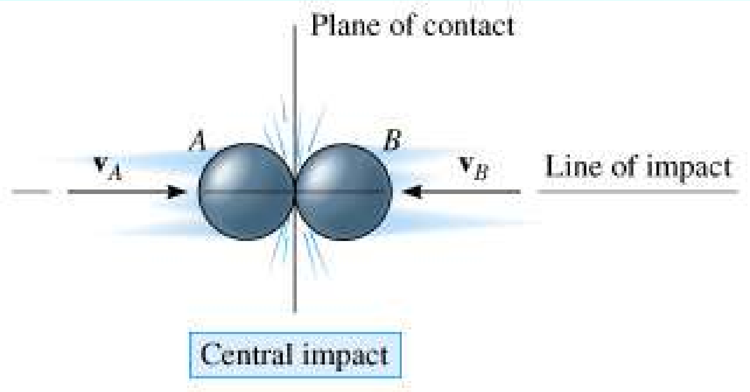
- Next Monday you will have the first 15 minutes of the lecture available to fill out your online Course Evaluation of the course and instructor
- You can also complete the evaluation at any time you wish (until Dec. 3) from Canvas -> Phys170.101 -> Course Evaluation
- ME Homework 12 (the last one, released this evening), is due Sunday December 2, 11:59 PM
- Final exam review sessions next week in tutorial slots

Lecture Outline/Learning Goals

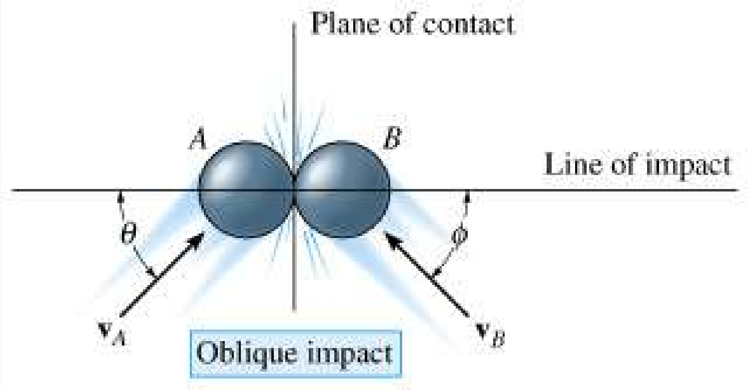
- 15.4 Review / finish Central Impact
- 15.4 Oblique Impact (including some results/equations not discussed in text)
- Worked example of oblique impact

15.4 Impact

- **IMPACT (defn):** Collision (interaction) of two bodies that takes place over a very short time, and which involves relatively large (impulsive) forces (examples: collisions of billiard balls, hammer hitting a nail, baseball bat hitting a baseball …)



- **Central impact:** Direction of motion of centers of mass (geometric centers assuming constant density) of particles is along a line that passes through both centers (this line is called the **line of impact**)



- **Oblique impact:** Direction of motion of one or both of the particle centers of mass is at an angle with the line of impact

- Also note the identification of the **plane of contact** in the figures, which is always perpendicular to the line of impact

Central Impact (continued)

- We are thus led to consider the **ratio** of the restitution impulse to the deformation impulse in an impact – a quantity known as the **coefficient of restitution,** denoted by e, and defined by

$$e = \frac{\int R\,dt}{\int P\,dt}$$

- From the point of view of problem solving, the importance of the coefficient of restitution is that it provides a relation between the initial and final velocities of the particles as follows

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{\text{relative velocity after impact}}{\text{relative velocity before impact}}$$

(We will not derive this formula here – see the text if you are interested in the derivation)

Central Impact (continued)

- Thus, provided that the coefficient of restitution, e, for a collision is specified, then given the initial particle velocities, the above equation along with the equation for conservation of linear momentum can be simultaneously solved to determine the final particle velocities

- Also note that another typical type of problem that can be solved using the coefficient of restitution involves an object such as a ball, which is dropped from some height on to the ground, and given e, one is supposed to calculate how high the ball rebounds. In such cases, conservation of momentum does not usually help in solving the problem (why?), but an expression such as

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{0 - (v_A)_2}{(v_A)_1 - 0} = \frac{-(v_A)_2}{(v_A)_1} \quad \rightarrow \quad (v_A)_2 = -e\,(v_A)_1$$

can be used to determine the velocity, $(v_A)_2$, of the ball immediately after rebound, in terms of the velocity, $(v_A)_1$, just before impact and e

Central Impact (continued)

- From the definition

$$e = \frac{\int R dt}{\int P dt}$$

and from our previous observation that in real collisions $\int P dt > \int Q dt$ with equality being achieved only in the idealized case of an **elastic** collision, we conclude that the coefficient of restitution must satisfy

$$0 \le e \le 1$$

- The two limiting cases have special physical interpretations that should be noted

Central Impact (continued)

Elastic impact ($e = 1$)

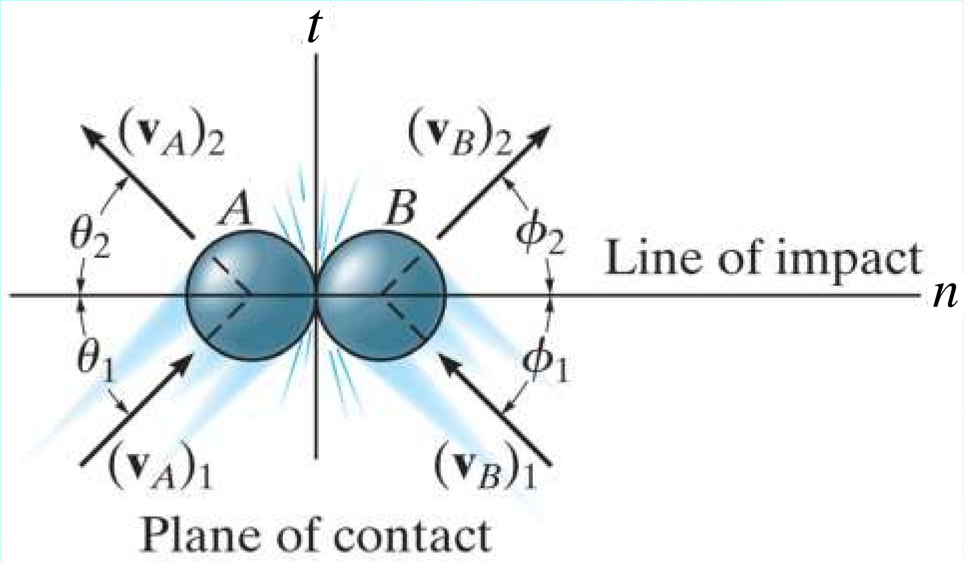
- We have already noted this (ideal) case when the deformation and restoration impulses are the same, and that the collision is termed **elastic** in this instance

- It is important to note that, in this case, **the total kinetic energy of the particles will be conserved by the collision** (assuming that no other forces that do work on the particles act during the collision) – this is the **only** value of e for which kinetic energy is conserved

Plastic impact ($e = 0$)

- In this case there is **no** restitution impulse after maximum deformation of the particles, and the particles thus **stick together** and have a common velocity following the collision – in this type of collision a maximal amount of kinetic energy will be dissipated (via various mechanisms) during the deformation phase

Oblique Impact



- Establish *n* axis (normal axis) along the line of impact and *t* axis (transverse axis) within plane of contact

- **Impulsive forces of deformation and restitution act *only in the n direction***

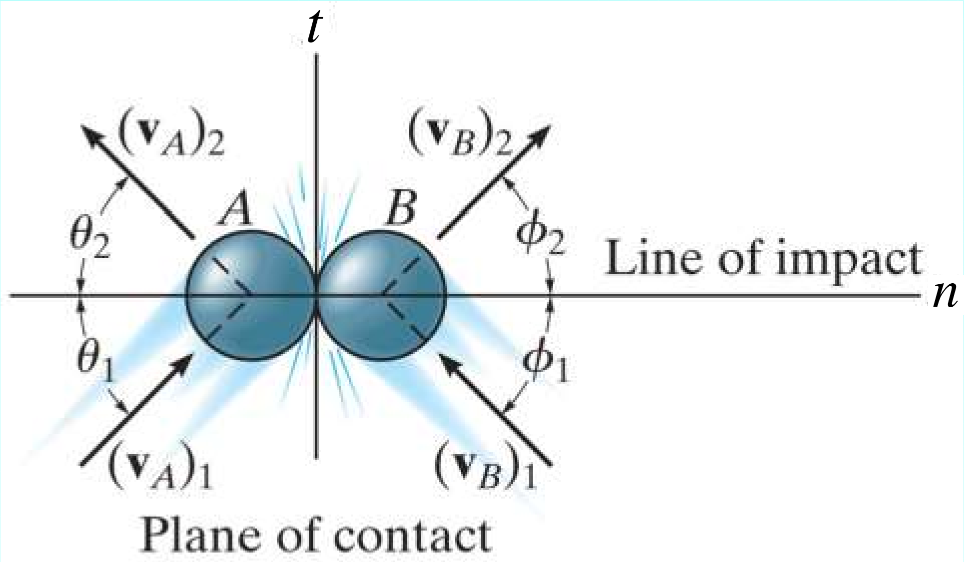
- Momentum is conserved along the line of impact, so that (sums are over particles)

$$\sum m v_{1n} = \sum m v_{2n}$$

- The coefficient of restitution relates the relative-velocity components of the particles along the *n axis*

$$e = \frac{v_{B2n} - v_{A2n}}{v_{A1n} - v_{B1n}}$$

Oblique Impact



- Along the *t* axis, perpendicular to the line of impact, the momenta of particles *A* and *B* are individually conserved since no impulses act on either of them in this direction

  Thus we have

$$m_A v_{A2t} = m_A v_{A1t}$$
$$m_B v_{B2t} = m_B v_{B1t}$$

or

$$v_{A2t} = v_{A1t}$$
$$v_{B2t} = v_{B1t}$$

- A typical problem involving oblique impact will require us to find the final velocities of two particles given the initial velocities and the coefficient of restitution, e

- We can manipulate the equations above to derive some additional equations that are useful for this purpose

- Let us start with the equation for conservation of momentum in the n direction

$$m_A v_{A1n} + m_B v_{B1n} = m_A v_{A2n} + m_B v_{B2n}$$

and the relation between e and the velocity differences

$$e = \frac{v_{B2n} - v_{A2n}}{v_{A1n} - v_{B1n}}$$

- Solve the last equation for v_{B2n}

$$v_{B2n} = v_{A2n} + e\left(v_{A1n} - v_{B1n}\right)$$

and substitute it in the first equation

$$m_A v_{A1n} + m_B v_{B1n} = (m_A + m_B) v_{A2n} + e m_B (v_{A1n} - v_{B1n})$$

- This can be written as

$$(m_A - e m_B) v_{A1n} + (1 + e) m_B v_{B1n} = (m_A + m_B) v_{A2n}$$

and solving for v_{A2n} we have

$$v_{A2n} = \frac{(m_A - e m_B) v_{A1n} + m_B (1 + e) v_{B1n}}{m_A + m_B}$$

- Similarly, for v_{B2n} we find

$$v_{B2n} = \frac{(m_B - e m_A) v_{B1n} + m_A (1 + e) v_{A1n}}{m_B + m_A}$$

- Summarizing: Given the masses m_A , m_B , the coefficient of restitution, e, and the initial velocity components v_{A1n} , v_{A1t} , v_{B1n} and v_{B1t} , where n is the (normal) axis along the line of action, and t is the (transverse) axis perpendicular to n, the final velocity components are given by

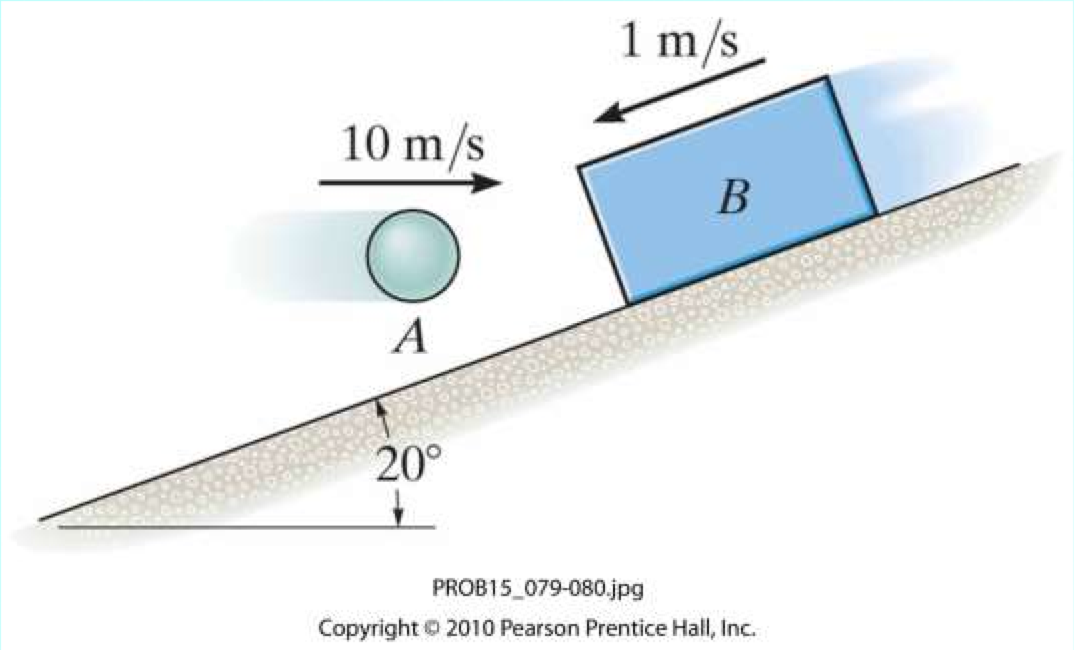
$$v_{A2n} = \frac{(m_A - em_B)v_{A1n} + m_B(1+e)v_{B1n}}{m_A + m_B}$$

$$v_{A2t} = v_{A1t}$$

$$v_{B2n} = \frac{(m_B - em_A)v_{B1n} + m_A(1+e)v_{A1n}}{m_B + m_A}$$

$$v_{B2t} = v_{B1t}$$

Problem 15-79 (Page 260, 12th edition)

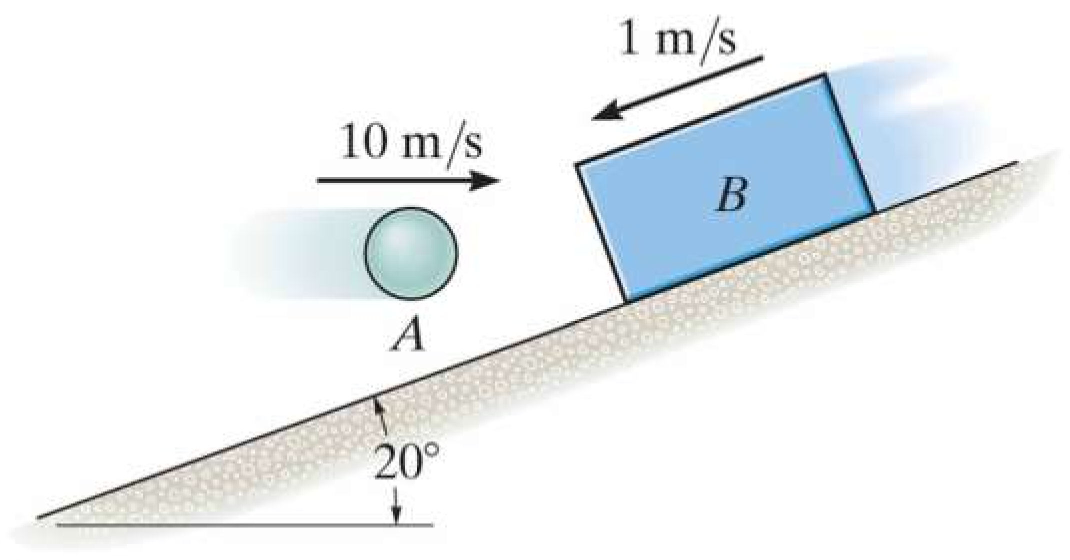


The 2 kg ball travels horizontally at 10 m/s when it strikes the 6 kg block traveling down the inclined plane at 1 m/s. The coefficient of restitution for the collision is 0.6.

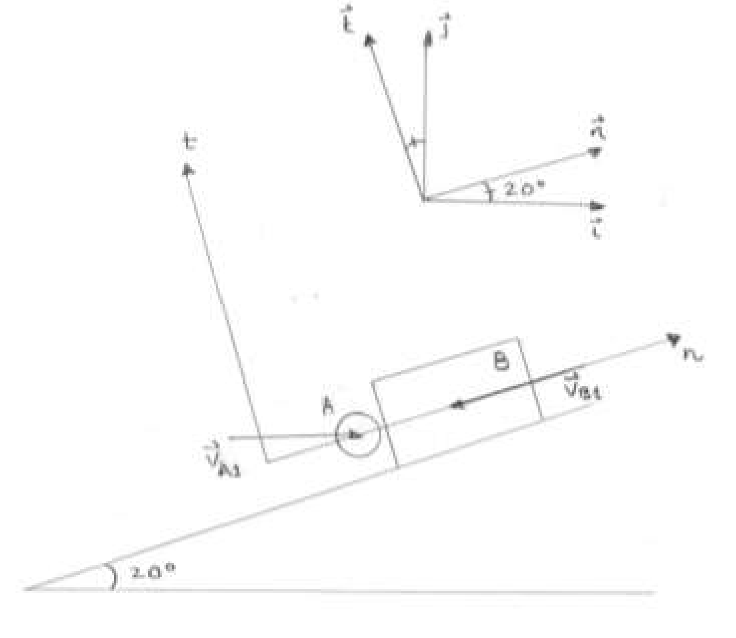
(1) Determine the velocities of the ball and the block just after impact

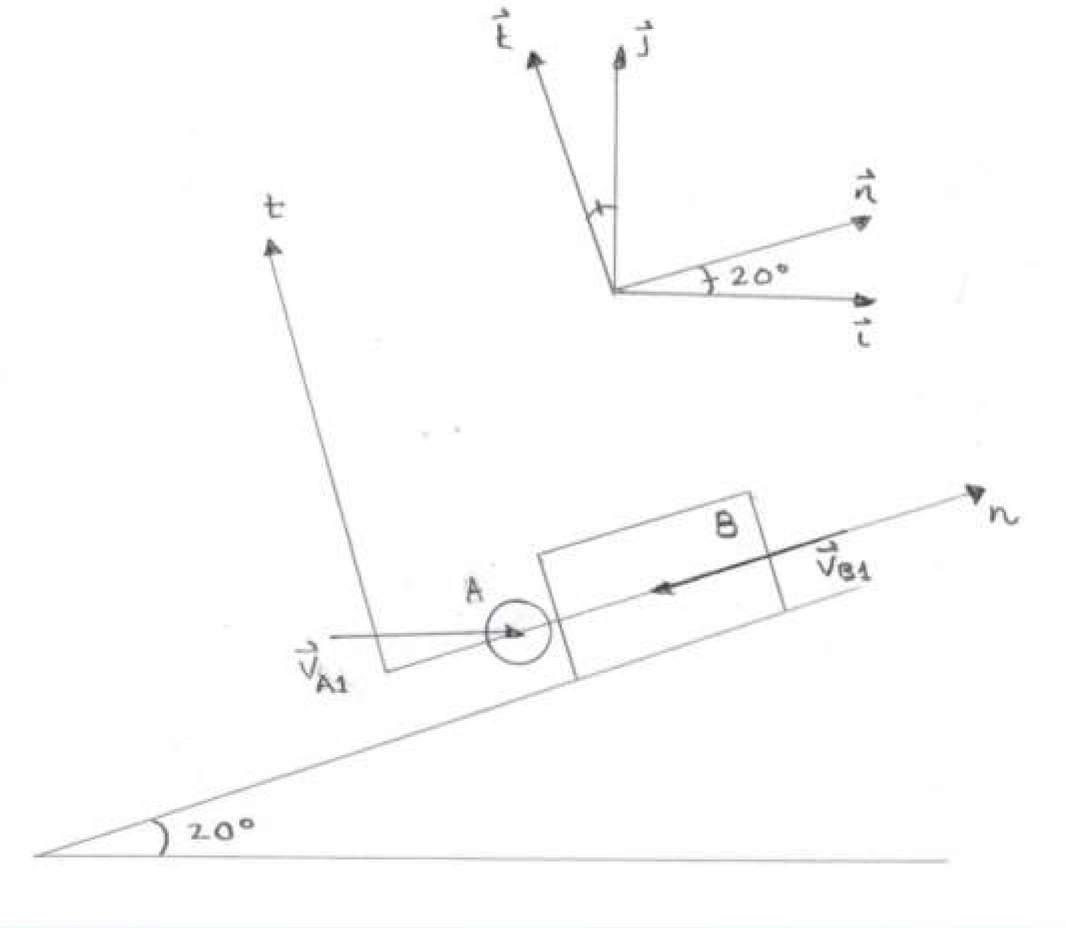
(2) Express the velocity of the ball just after impact as a Cartesian vector in terms of its speed and angle with the negative *x*-axis

(3) Determine the distance *B* slides up the plane before it momentarily stops. The coefficient of kinetic friction between the block and the plane is 0.4



PROB15_079-080.jpg





Solution strategy

Write down initial velocities in *nt* coordinates

Compute final velocity components using equations derived above

Express final velocity of ball as a vector in Cartesian (*xy*) coords.

Compute distance ball travels up ramp post-collision using energy methods

Data

$$m_A = 2 \text{ kg} \qquad m_B = 6 \text{ kg} \qquad e = 0.6$$

$$\vec{v}_{A1} = 10\vec{i} \text{ m/s}$$

$$\vec{v}_{B1} = (-\cos 20^\circ \vec{i} - \sin 20^\circ \vec{j}) \text{ m/s}$$

Initial velocities (nt components, suppressing units)

$$\vec{v}_{A1} = 10(\cos 20^\circ \vec{n} - \sin 20^\circ \vec{t})$$

$$\vec{v}_{B1} = -\vec{n}$$

Final velocity components

$$v_{A2t} = v_{A1t} = -10\sin 20^{\circ} = -3.420$$

$$v_{B2t} = v_{B1t} = 0$$

$$v_{A2n} = \frac{(m_A - em_B)v_{A1n} + m_B(1+e)v_{B1n}}{m_A + m_B} = \frac{[2-0.6(6)]10\cos 20^{\circ} + 6(1+0.6)(-1)}{2+6}$$

$$= -3.079$$

$$v_{B2n} = \frac{(m_B - em_A)v_{B1n} + m_A(1+e)v_{A1n}}{m_B + m_A} = \frac{[6-0.6(2)](-1) + 2(1+0.6)10\cos 20^{\circ}}{6+2}$$

$$= 3.159$$

Final velocities

$$\vec{v}_{A2} = -3.08\vec{n} - 3.42\vec{t}$$

Express this in terms of a magnitude and angle w.r.t. the negative *n* axis

$$\vec{v}_{A2} = 4.60(-\cos 48.0^{\circ}\,\vec{n} - \sin 48.0\vec{t}\,) \text{ m/s}$$

$$\vec{v}_{B2} = 3.16\,\vec{n} \text{ m/s}$$

Velocity of the ball after impact as a Cartesian vector (angle w.r.t. negative x axis)

$$\vec{v}_{A2} = 4.60(-\cos 68.0^{\circ}\,\vec{i} - \sin 68.0^{\circ}\,\vec{j}\,) \text{ m/s}$$

How far does block slide up the plane?

Data

$m_B = 6$ kg $\qquad \theta = 20^\circ \qquad \mu = 0.4$

$\vec{v}_{B2} = 3.16\vec{n}$ m/s $\qquad h_{B2} = 0$

$\vec{v}_{B3} = 0 \qquad h_{B3} = d\sin\theta$

Energy balance equation (d is distance block slides)

$$\frac{1}{2} m_B v_{B2}^2 - \mu N d = m_B g h_{B3} = m_B g d \sin\theta \qquad (1)$$

Equilibrium in direction perpendicular to plane

$$N = m_B g \cos\theta \qquad (2)$$

Substitute (2) in (1) and solve for d

$$d = \frac{v_{B2}^2}{2g(\sin\theta + \mu\cos\theta)} = \frac{(3.16)^2}{2(9.81)(\sin 20^\circ + 0.4\cos 20^\circ)}$$

$$= 0.709 \text{ m}$$