PHYS 170 Section 101

Lecture 30

November 21, 2018

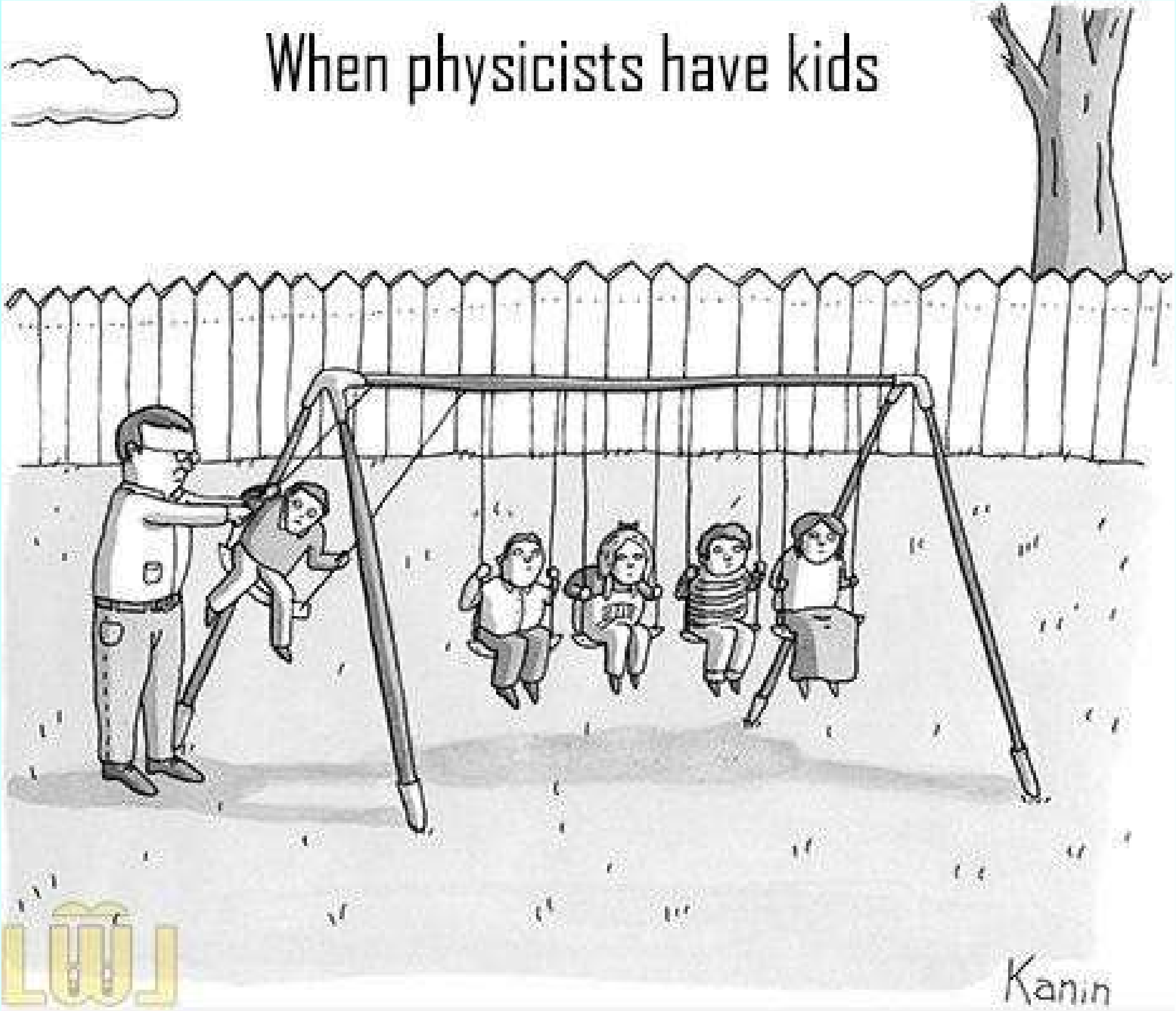
Pseudo-announcement—Nov 21

- Do you want final review sessions next Tuesday, in lieu of tutorials?

Lecture Outline/Learning Goals

- Worked example using conservation of linear momentum
- 15.4 Impact

Conservation of Linear Momentum


When physicists have kids
Kanin

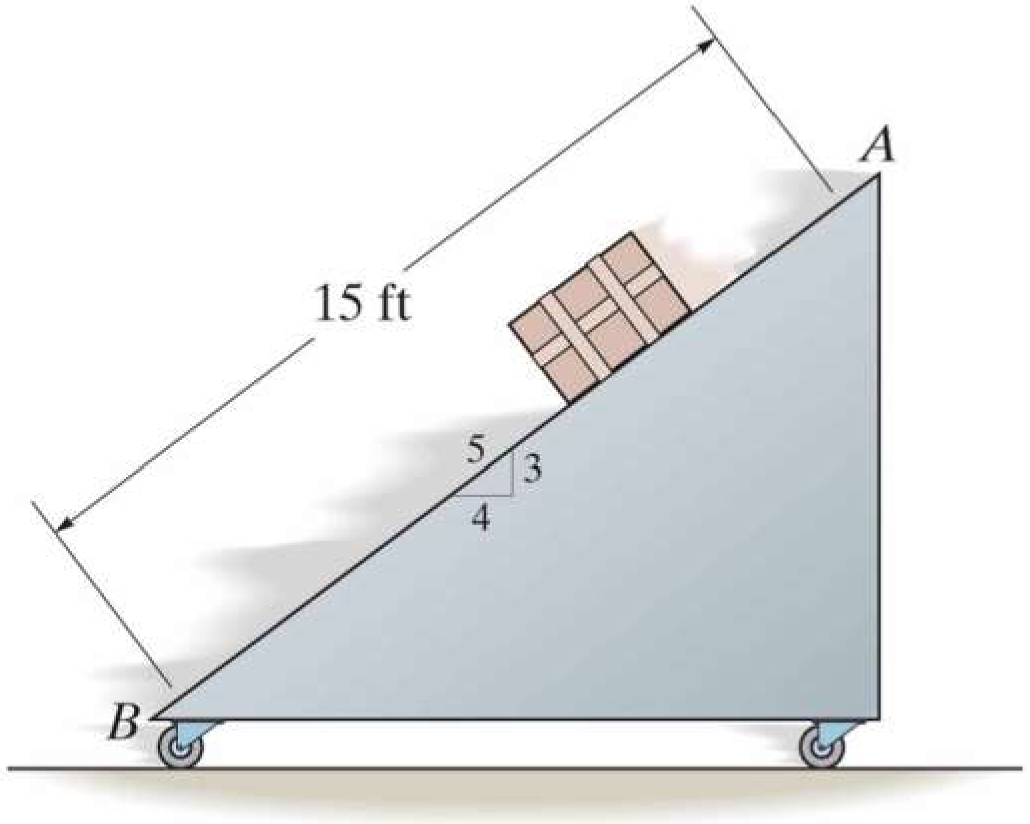
Problem 15-47 (Page 246, 12th edition)

The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to *B* from rest at *A*.

(1) Determine the speed of the ramp when the crate reaches *B*

(2) Determine the velocity of the crate when it reaches *B*. Express the velocity as a Cartesian vector in terms of the crate's speed and the angle the velocity makes with the horizontal

(3) Determine the kinetic energies of the ramp and the crate when the crate reaches *B*



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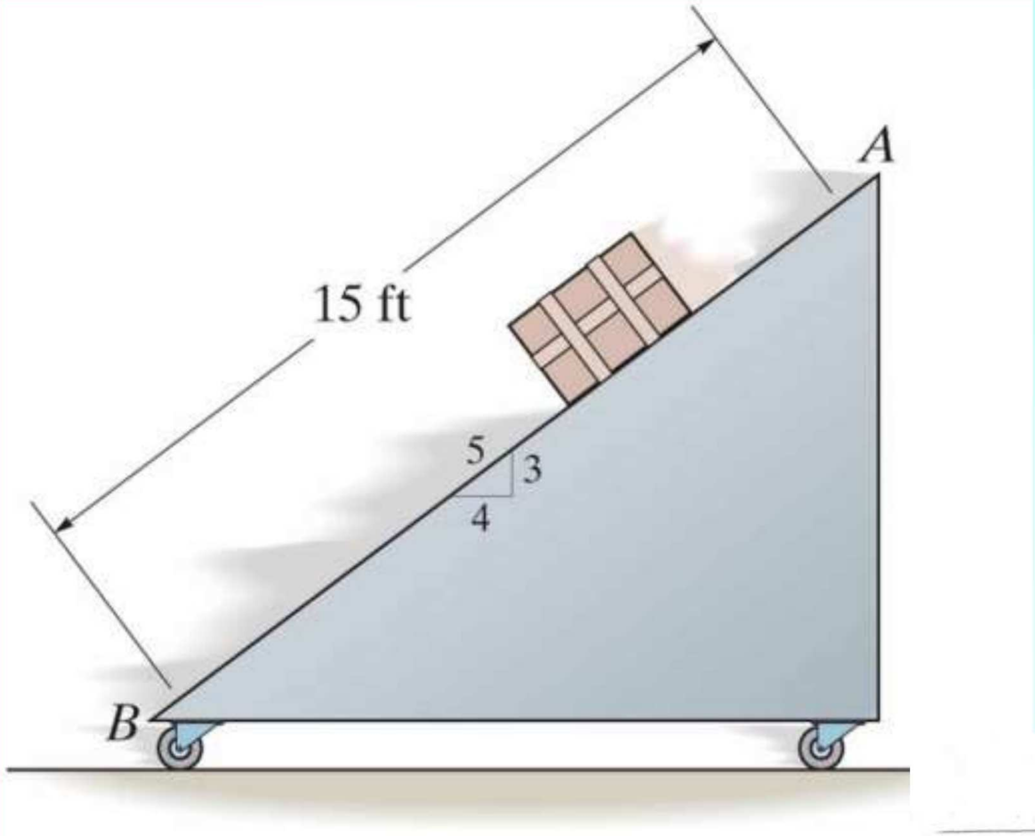

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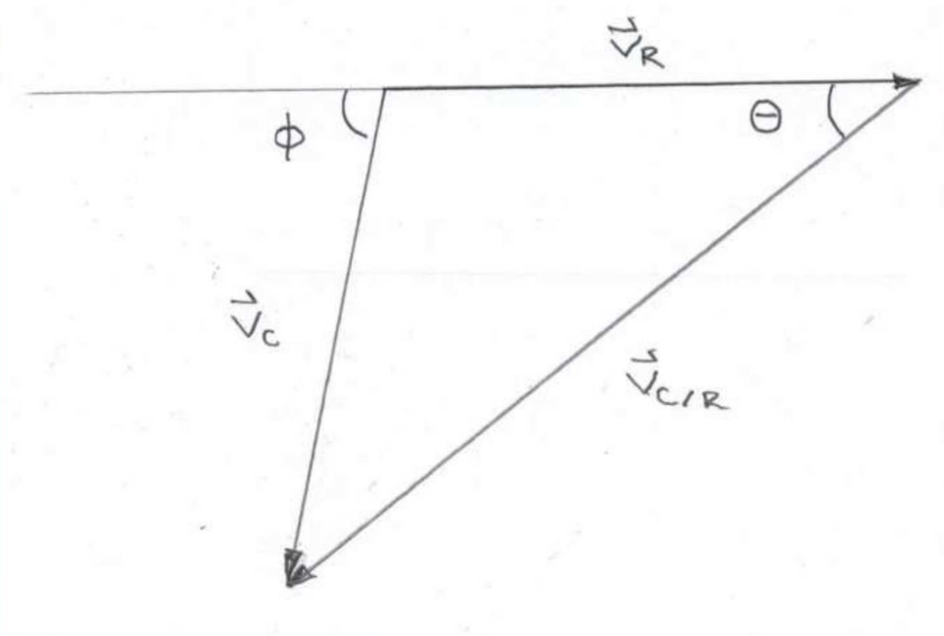
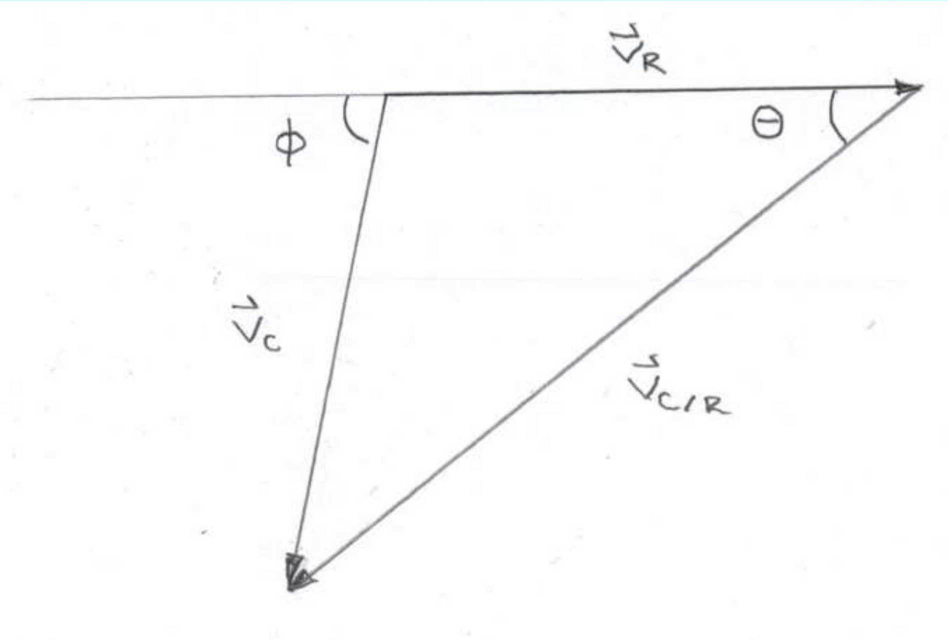
Velocity triangle

$$\vec{v}_C = \vec{v}_R + \vec{v}_{C/R}$$

$$\theta = \tan^{-1}(3/4) = 36.9^{\circ}$$





Solution strategy

Write down equations for:

(1) Conservation of energy

(2) Conservation of total linear momentum in the horizontal direction

(3) Relative motion

Solve equations for v_R, v_C, $v_{C/R}$ and ϕ

Determine velocities in Cartesian coordinates when the crate is at *B*

Determine kinetic energies when the crate is at *B*

Data

$$W_R = m_R g = 120 \text{ lb} \qquad W_C = m_C g = 80 \text{ lb} \qquad g = 32.2 \text{ ft/s}^2$$

$$\theta = \tan^{-1}(3/4) = 36.9^\circ \qquad h_C = (3/5)\cdot 15 \text{ ft} = 9 \text{ ft}$$

Velocities

$$\vec{v}_R = v_R \vec{i}$$

$$\vec{v}_C = v_C(-\cos\phi\,\vec{i} - \sin\phi\,\vec{j})$$

$$\vec{v}_{C/R} = v_{C/R}(-\cos\theta\,\vec{i} - \sin\theta\,\vec{j})$$

Unknowns

$$v_R,\ \mathrm{v}_C,\ v_{C/R},\ \phi$$

Need four equations to determine the four unknowns

Conservation of energy

$$\frac{1}{2}m_R v_R^2 + \frac{1}{2}m_C v_C^2 = m_C g h_C \qquad (1)$$

Conservation of total linear momentum in the horizontal direction

(note that v_C is a positive quantity, and that the initial total momentum is 0)

$$m_R v_R - m_C v_C \cos\phi = 0 \qquad (2)$$

Relative motion (refer back to expressions for velocities)

$$\vec{v}_C = \vec{v}_R + \vec{v}_{C/R}$$

Vertical component

$$-v_C \sin\phi = -v_{C/R} \sin\theta \qquad (3)$$

Relative motion

$$\vec{v}_C = \vec{v}_R + \vec{v}_{C/R}$$

Horizontal component

$$-v_C \cos\phi = v_R - v_{C/R} \cos\theta \qquad (4)$$

Solve equations (1)-(4) for the four unknowns

Write $v_{C/R}$ in terms of v_R

From (4) we have

$$v_{C/R} = \frac{v_R + v_C \cos\phi}{\cos\theta}$$

From (2) we have

$$\mathrm{v}_C \cos\phi = \frac{m_R v_R}{m_C}$$

Substituting the rhs of the above into the previous expression we have

$$v_{C/R} = \frac{v_R + \dfrac{m_R v_R}{m_C}}{\cos\theta} = \frac{(m_R + m_C)v_R}{m_C \cos\theta} = \frac{5}{4}\left(\frac{200}{80}\right)v_R = \frac{25}{8}v_R \qquad (5)$$

Write v_C in terms of v_R

From (3) and (5)

$$v_C \sin\phi = v_{C/R}\sin\theta = \frac{3}{5}\left(\frac{25}{8}\right)v_R = \frac{15}{8}v_R$$

$$v_C \cos\phi = -v_R + v_{C/R}\cos\theta = -v_R + \left(\frac{4}{5}\right)\left(\frac{25}{8}v_R\right) = \frac{3}{2}v_R$$

Square the last two results and sum them

$$v_C{}^2 \sin^2\phi + v_C{}^2 \cos^2\phi = v_C^2 = \left(\frac{15}{8}\right)^2 v_R^2 + \left(\frac{3}{2}\right)^2 v_R^2$$

$$v_C^2 = \frac{369}{64} v_R^2 \qquad (6)$$

Now substitute (6) in (1)

$$\frac{1}{2}\left(\frac{120}{32.2}\right)v_R^2 + \frac{1}{2}\left(\frac{80}{32.2}\right)v_C^2 = 80(9) \quad \Rightarrow \quad v_R = 8.93 \text{ ft/s}$$

Now consider equations (3) and (4)

$$-v_C \sin\phi = -v_{C/R} \sin\theta$$

$$-v_C \cos\phi = v_R - v_{C/R} \cos\theta$$

Dividing these two we have

$$\tan\phi = \frac{v_{C/R} \sin\theta}{v_{C/R} \cos\theta - v_R}$$

so

$$\phi = \tan^{-1}\left(\frac{v_{C/R} \sin\theta}{v_{C/R} \cos\theta - v_R}\right) = \tan^{-1}\left(\frac{\left(\frac{25}{8} v_R\right)\left(\frac{3}{5}\right)}{\left(\frac{25}{8} v_R\right)\left(\frac{4}{5}\right) - v_R}\right) = \tan^{-1}\left(\frac{25(3)}{(25)(4) - 40 v_R}\right) = 51.3^\circ$$

Can now compute v_C and $v_{C/R}$

$$v_C = \frac{\frac{15}{8} v_R}{\sin\phi} = \frac{\left(\frac{15}{8}\right)(8.93)}{\sin 51.3^\circ} = 21.4 \text{ ft/s}$$

$$v_{C/R} = \frac{25}{8} v_R = \frac{25}{8}(8.93) = 27.9 \text{ ft/s}$$

Velocities when the crate is at B

$$\vec{v}_R = 8.93\vec{i} \text{ ft/s}$$

$$\vec{v}_C = 21.4\left(-\cos 51.3^\circ \vec{i} - \sin 51.3^\circ \vec{j}\right) \text{ ft/s}$$

$$\vec{v}_{C/R} = 27.9(-\cos 36.9^\circ \vec{i} - \sin 36.9^\circ \vec{j}) \text{ ft/s}$$

Note that the crate slides down the ramp at an angle $\phi = 51.3^\circ$ to the horizontal. This is steeper than the plane angle $\theta = 36.9^\circ$ because the ramp rolls to the right while the crate slides to the left and down the ramp.

Kinetic energies when the crate is at *B*

$$\frac{1}{2}m_R v_R^2 = \frac{1}{2}\left(\frac{120}{32.2}\right)(8.93)^2 \text{ ft}\cdot\text{lb} = 149 \text{ ft}\cdot\text{lb}$$

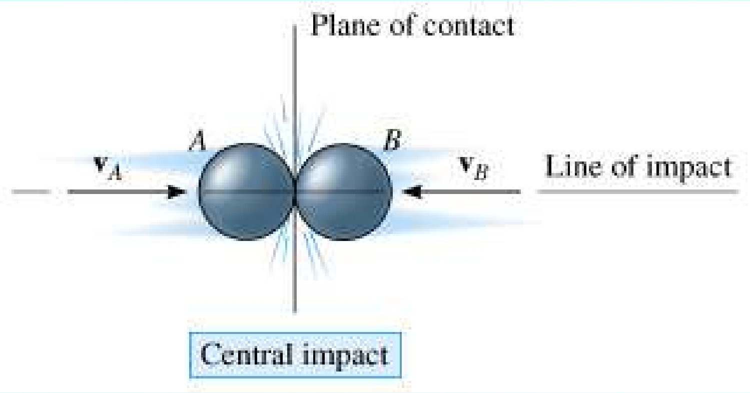
$$\frac{1}{2}m_C v_C^2 = \frac{1}{2}\left(\frac{80}{32.2}\right)(21.4)^2 \text{ ft}\cdot\text{lb} = 571 \text{ ft}\cdot\text{lb}$$

The total kinetic energy is equal to the initial potential energy

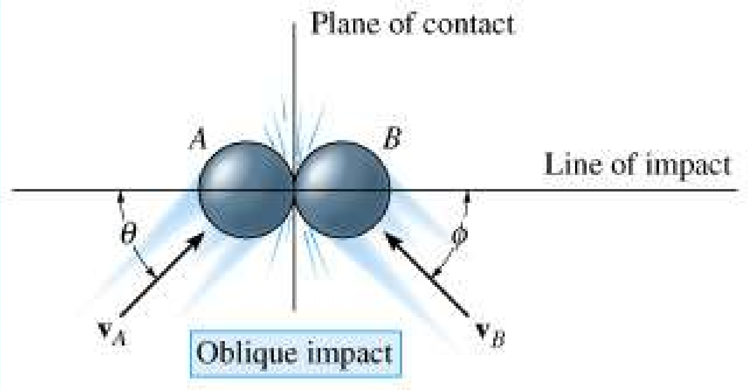
$$m_C g h_C = 80(9) \text{ ft}\cdot\text{lb} = 720 \text{ ft}\cdot\text{lb}$$

15.4 Impact

- **IMPACT (defn):** Collision (interaction) of two bodies that takes place over a very short time, and which involves relatively large (impulsive) forces (examples: collisions of billiard balls, hammer hitting a nail, baseball bat hitting a baseball …)



- **Central impact:** Direction of motion of centers of mass (geometric centers assuming constant density) of particles is along a line that passes through both centers (this line is called the **line of impact**)

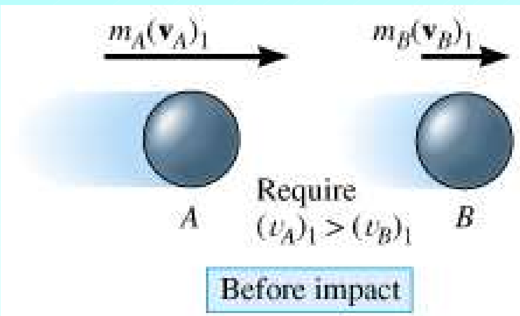


- **Oblique impact:** Direction of motion of one or both of the particle centers of mass is at an angle with the line of impact

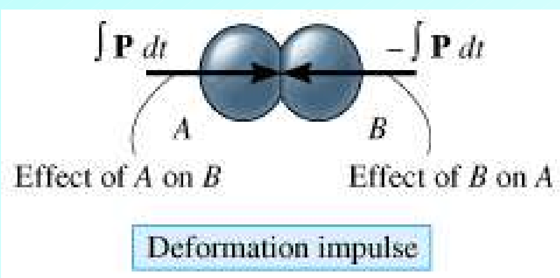
- Also note the identification of the **plane of contact** in the figures, which is always perpendicular to the line of impact

Central Impact

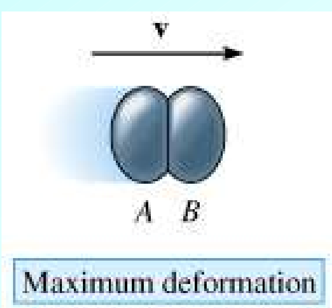
- In order to better understand the basics physics underlying impact, as well as to motivate the equations that we will use in its analysis, it is useful to consider the following 5 phases of the central impact of two smooth particles



1. **Before impact:** Here, the particles have initial momenta $m_A(\mathbf{v}_A)_1$ and $m_B(\mathbf{v}_B)_1$ as shown. In order for a collision to occur, we must have $(v_A)_1 > (v_B)_1$

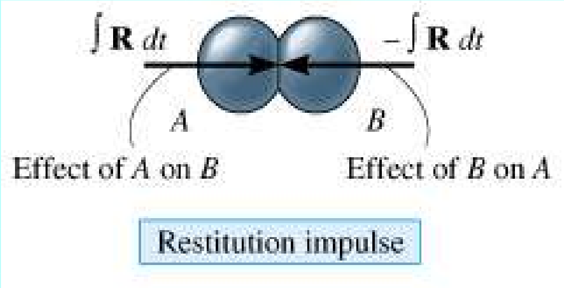


2. **Deformation impulse:** During the actual collision, the particles must be viewed as being **deformable** or non-rigid. This phase is characterized by a **period of deformation** in which the particles exert equal but opposite deformation impulses $\int P dt$ on each other

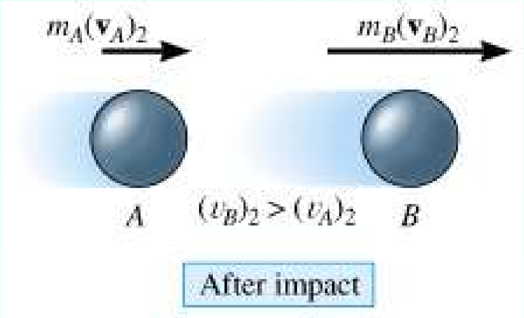


3. **Maximum deformation:** At the instant when the particles are maximally deformed (and only at this instant in general), the particles will be moving with the same velocity, $\mathbf{v}$

Central Impact (continued)



4. **Restitution impulse:** This is a period of restitution, wherein the bodies will either regain their original shapes, or remain permanently deformed. As with the deformation stage, during this phase the particles exert equal and opposite restitution impulses $\int Rdt$ on each other, and these impulses tend to push the bodies apart. In any real collision one finds that the deformation impulse exceeds the restitution impulse, i.e. that $\int Pdt > \int Rdt$. In the idealized case that the deformation and restitution impulses are identical, the collision is called **elastic**



5. **After impact:** Assuming that the particles **do** separate after the collision (i.e. that they do not stick together), then just after separation they will have final momenta $m_A(\mathbf{v}_A)_2$ and $m_B(\mathbf{v}_B)_2$, and the particle velocities will satisfy $(v_B)_2 > (v_A)_2$

Central Impact (continued)

- The typical problem involving impact is to find the final velocities $(v_A)_2$ and $(v_B)_2$ of the particles given the initial velocities $(v_A)_1$ and $(v_B)_1$ and other problem parameters

- Treating the two particles as a system, and observing that the deformation and restitution impulses are **internal** (and thus must cancel – i.e. must be equal and opposite as already noted), we know that one equation that we will have at our disposal will be conservation of linear momentum for the system:

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

- Since we have two unknowns in general (observe that each velocity for central impact reduces to a single [scalar] quantity), we need another equation

- It is clear that this equation should have something to do with the nature of the collision – and in particular, on the relative strengths of the deformation and restitution impulses

Central Impact (continued)

- We are thus led to consider the **ratio** of the restitution impulse to the deformation impulse in an impact – a quantity known as the **coefficient of restitution,** denoted by e, and defined by

$$e = \frac{\int R dt}{\int P dt}$$

- From the point of view of problem solving, the importance of the coefficient of restitution is that it provides a relation between the initial and final velocities of the particles as follows

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{\text{relative velocity after impact}}{\text{relative velocity before impact}}$$

(We will not derive this formula here – see the text if you are interested in the derivation)