PHYS 170 Section 101
Lecture 22
October 29, 2018

Lecture Outline/Learning Goals

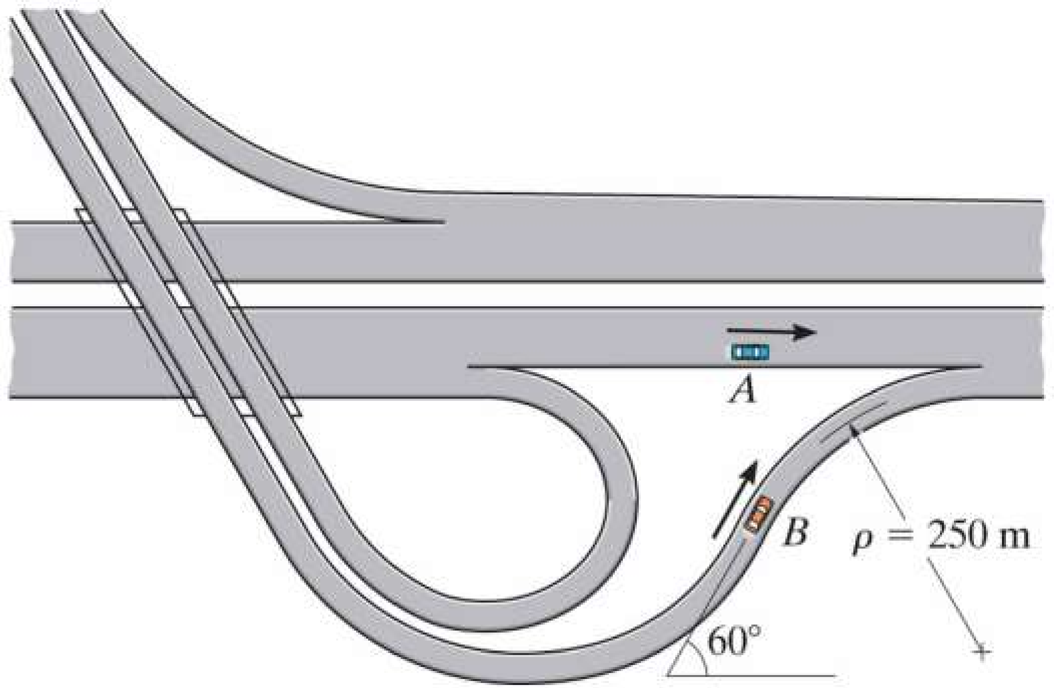
- Finish worked example of relative motion
- End of material for second midterm
- Start Ch. 13—Kinetics of a Particle Force and Acceleration
- 13. 1 Newton’s Second Law of Motion
- 13.2 The Equation of Motion
- 13.4 Equations of Motion: Rectangular Coordinates
- Worked example of equations of motion in rectangular coordinates

Problem 12-228 (page 100, 12th edition)

At the instant shown, car *A* travels east along the highway at 30 m/s and accelerates at 2 m/s^2.

At the same instant, car *B* travels on the interchange curve at 15 m/s and decelerates at 0.8 m/s^2.

(1) Determine the velocity and acceleration of *B* relative to *A* at this instant.



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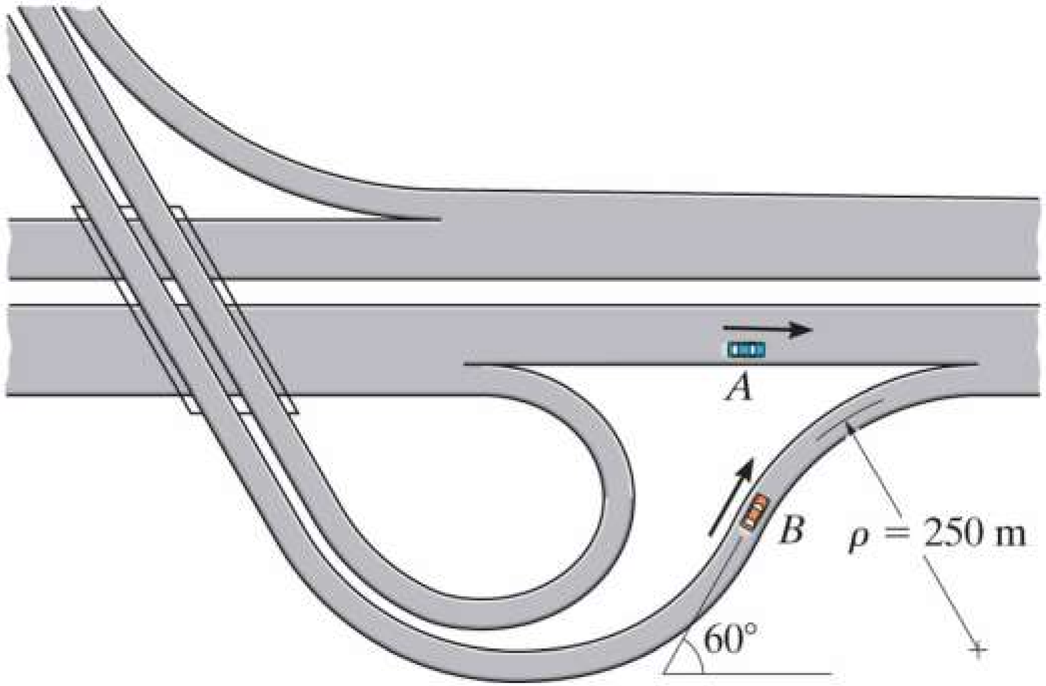

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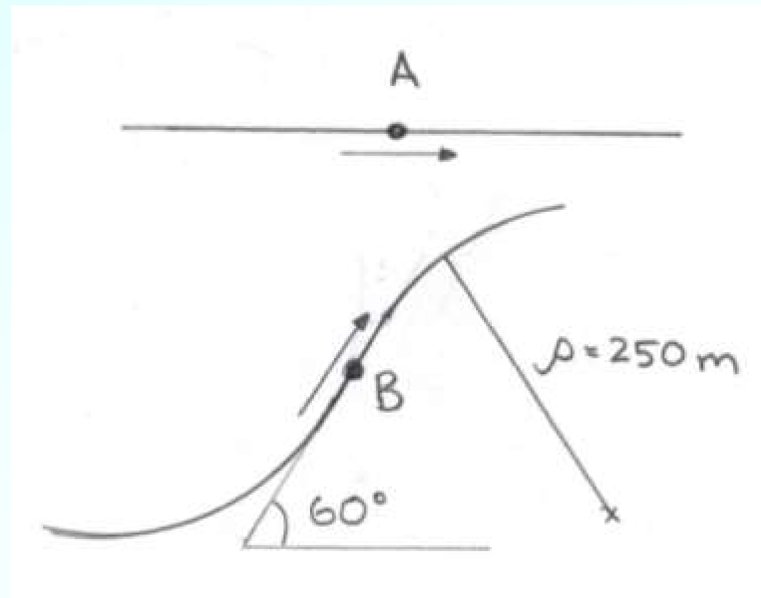
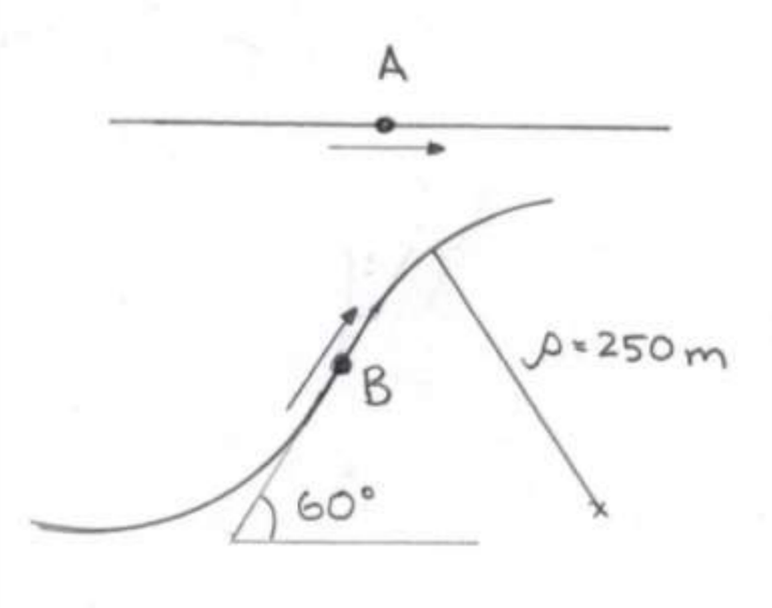




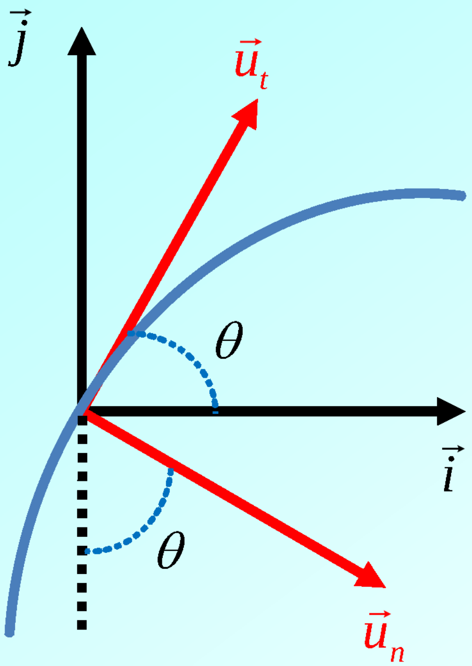
Solution strategy

Express velocities and accelerations in Cartesian components and then use

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$
$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

For car B, express velocity and acceleration in terms of tangential and normal components, and then express $\vec{u}_t$ and $\vec{u}_n$ in terms of $\vec{i}$, $\vec{j}$ and θ

Relationship between tangential/normal and Cartesian unit vectors



$$\vec{u}_t = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$
$$\vec{u}_n = \sin\theta\,\vec{i} - \cos\theta\,\vec{j}$$

Warning!! These are NOT general formulae. They are specific to the particular orientation of the two sets of unit vectors. You need to be able to derive equivalent relationships for other orientations.

Velocities and accelerations ($\vec{i}$ points east, $\vec{j}$ points north)

$$\vec{v}_A = v_A \vec{i} \qquad \vec{v}_B = v_B \vec{u}_t$$

$$\vec{a}_A = a_A \vec{i} \qquad \vec{a}_B = \dot{v}_B \vec{u}_t + \frac{v_B^2}{\rho} \vec{u}_n$$

$$\vec{u}_t = \cos\theta \vec{i} + \sin\theta \vec{j} \qquad \vec{u}_n = \sin\theta \vec{i} - \cos\theta \vec{j}$$

$$v_A = 30 \text{ m/s} \qquad v_B = 15 \text{ m/s}$$

$$\dot{v}_A = 2 \text{ m/s}^2 \qquad \dot{v}_B = -0.8 \text{ m/s}^2$$

$$\rho = 250 \text{ m} \qquad \theta = 60^\circ$$

Velocity of B relative to A (Exercise: verify the calculations)

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = v_B(\cos\theta \vec{i} + \sin\theta \vec{j}) - v_A \vec{i}$$

$$= (-22.5\vec{i} + 13.0\vec{j}) \text{ m/s}$$

Acceleration of *B* relative to *A* (Exercise: verify the calculations)

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = \dot{v}_B(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}) + \frac{v_B^2}{\rho}(\sin\theta\,\vec{i} - \cos\theta\,\vec{j}) - \dot{v}_A\,\vec{i}$$

$$= (-1.62\vec{i} - 1.14\,\vec{j})\,\mathrm{m/s^2}$$

END OF MATERIAL FOR SECOND MIDTERM!!

Chapter 13: Kinetics of a Particle Force and Acceleration

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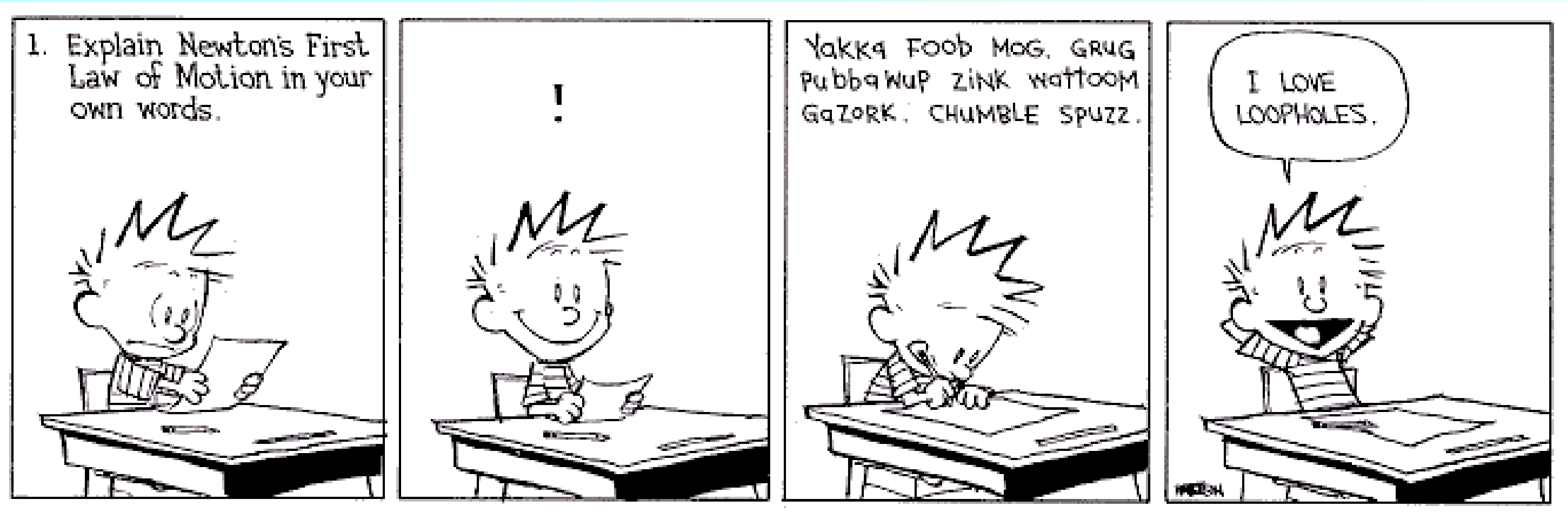
13_COC01

A car driving along this road will be subjected to forces that create both normal and tangential accelerations. In this chapter we will study how these forces are related to the accelerations they create.


1. Explain Newton's First Law of Motion in your own words.
!
Yakka Foob Mog. Grug Pubbawup Zink Wattoom Gazork. Chumble Spuzz.
I LOVE LOOPHOLES.

13.1 Newton's Laws of Motion (Again!)

- **FIRST LAW:** A particle originally at rest, or moving in a straight line with a constant velocity, will remain in this state provided that it is not subjected to an unbalanced force

- **SECOND LAW:** A particle acted upon by an unbalanced force **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force

- **THIRD LAW:** The mutual forces of action and reaction between two particles are equal, opposite and colinear

- In this chapter, we focus on applications of the second law, i.e. on the analysis of accelerated motion of particles (i.e. bodies that are idealized as particles), as well as the forces which cause the accelerated motion

- As previously, we will work in different coordinate systems

  - Rectangular (Cartesian) coordinates

  - Normal / tangential coordinates

  - Cylindrical (polar) coordinates

- Will generally restrict our problem solving to two dimensional problems

NEWTON'S 2nd LAW FOR A PARTICLE OF MASS *m* SUBJECTED TO A SINGLE UNBALANCED FORCE

$$\mathbf{F} = m\mathbf{a}$$

- As discussed previously in the course, we can view this equations as simultaneously defining

    1. What we mean by force, mass and acceleration

    2. The relationship between the three quantities

- The above equation is known as the **equation of motion**

MASS AND WEIGHT (review)

- **MASS:** Absolute (position-independent) attribute of particle that measures proportionality between applied force and acceleration, according to equation of motion

- **WEIGHT:** Non-absolute (position-dependent) attribute of particle which is the gravitational force of the earth acting on the particle

- For a particle of mass m, weight W is given by

$$W = mg$$

where g is known as the acceleration due to gravity, and is given by

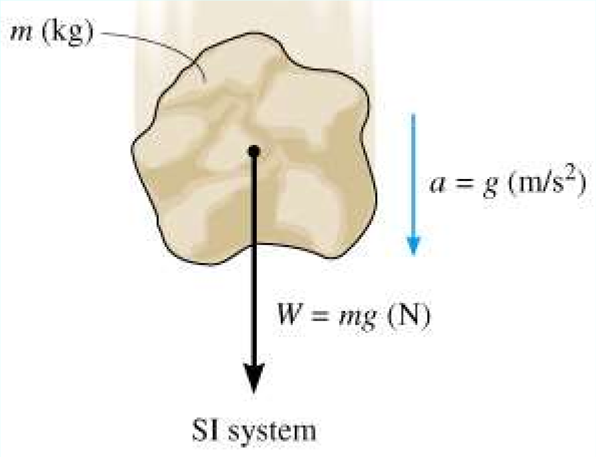
$$g = G\frac{m_{\text{earth}}}{r^2}$$

where m_{earth} is the mass of the earth and r is the distance between the particle and the center of the earth

MASS AND WEIGHT (continued)

- As usual, we will generally ignore the location-dependence of g, adopting a "standard" value of g in computations

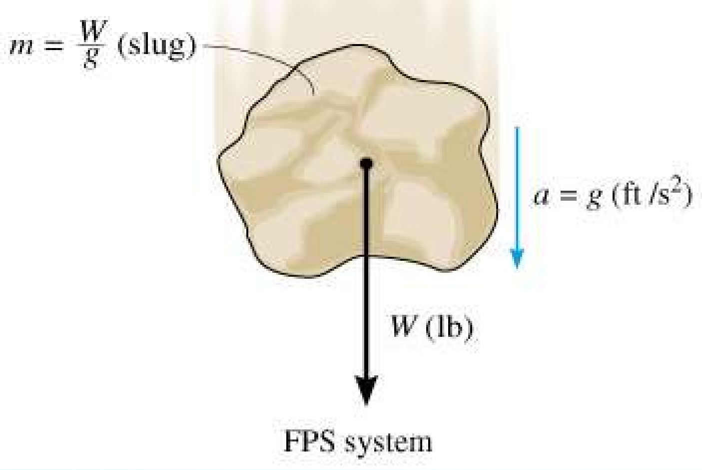
SI SYSTEM



- **MASS** is specified in kg
- Standard value of g : $9.81\ \mathrm{m/s^2}$
- Weight is measured in N and is computed from second law

$$W = mg \quad (\mathrm{N}) \qquad (g = 9.81\ \mathrm{m/s^2})$$

FPS SYSTEM



- **WEIGHT** is specified in lb
- Standard value of g: 32.2 ft / s^2
- Mass is measured in slugs and is computed from second law

$$m = \frac{W}{g} \quad \text{(slug)} \qquad (g = 32.2 \text{ ft / s}^2)$$

Be careful with FPS problems; will often need to compute mass of object from weight for use with second law

13.2 The Equation of Motion

- Consider the general case of a particle subjected to a system of forces with resultant $\mathbf{F}_R = \Sigma\mathbf{F}$

- The equation of motion is then

$$\boxed{\Sigma\mathbf{F} = m\mathbf{a}}$$

- We are familiar with free body diagrams; will continue to be of great use in our study of kinematics

Self study

INERTIAL FRAME OF REFERENCE

- As we have seen throughout the course, a key step in the solution of problems in mechanics is the establishment of a coordinate system (synonym: **frame**) with respect to which positions, velocities, accelerations, forces, … are defined and/or measured

- In order to apply the equation of motion in the form

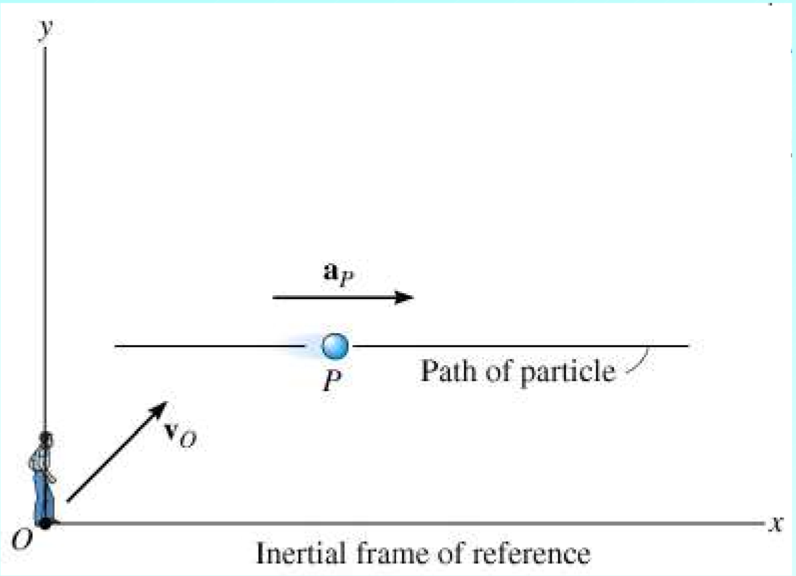
$$\Sigma \mathbf{F} = m\mathbf{a}$$

  it is crucial that the coordinate system we choose be **inertial** (or **Newtonian**), which, in a nutshell, means that the coordinate system is **not accelerating**

- Phrased another way, an inertial coordinate system is one which is
  1. Not rotating
  2. Either fixed, or translating in some direction with constant velocity

INERTIAL FRAME OF REFERENCE (continued)

Self study

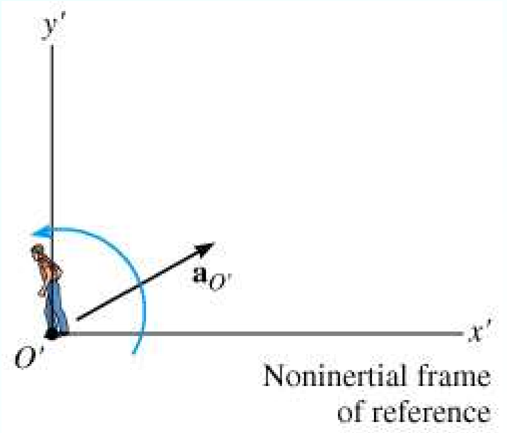


- **KEY POINT:** Observers at rest in different inertial coordinate systems will all agree on the acceleration $\mathbf{a}_P$ of a given particle as shown in the figure at the left

- **Conversely,** an observer in a non-inertial frame that has an acceleration $\mathbf{a}_{O'}$ relative to an inertial frame will measure the acceleration of the particle to be

$$\mathbf{a}_{P/O'} = \mathbf{a}_P - \mathbf{a}_{O'}$$

i.e. will measure the acceleration of the particle to be a **different** value



INERTIAL FRAME OF REFERENCE (continued)

Self study

- **KEY QUESTION:** How do we know when a coordinate system is inertial?
  - If we are given some **reference** inertial frame, this is easy to answer (in principle) – simply ensure that the frame in question is non-accelerating with respect to the reference frame
  - If we do **not** have access to a reference inertial frame, situation is trickier, and although very interesting, is beyond the scope of this course
- For our purposes, we adopt the first approach and note that even though the earth **is** rotating about its spin axis, and is in a roughly circular orbit about the sun, and that the solar system itself is accelerating with respect to the center of our galaxy …, the resulting accelerations can be neglected for the problems we consider
- Thus our working definition of an inertial frame will be one that is non-accelerating with respect to a coordinate system that is fixed to the surface of the earth

Self study

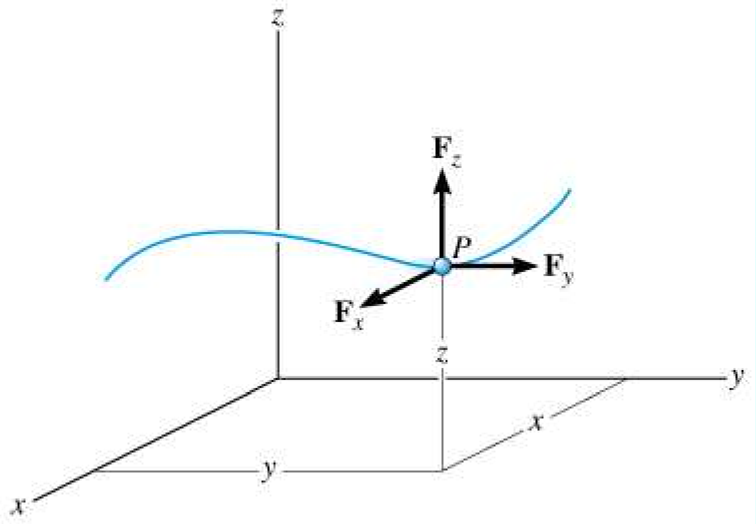
13.3 Equation of Motion for a System of Particles

- Will leave this section for self-study – will not discuss here since

    1. It is not relevant to the problems that we will consider

    2. It relies on the concept of the **center of mass** of a system of particles which we did not study in class

13.4 Equations of Motion: Rectangular Coordinates



- Consider a particle moving with respect to an inertial coordinate system (x,y,z) as shown in the figure

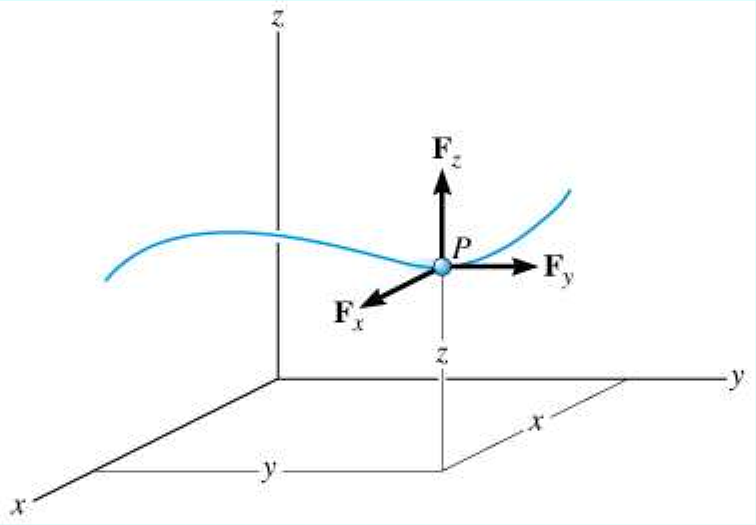
- Can then express (vector) forces acting on the particle, and the particle's (vector) acceleration in terms of **i**, **j** and **k** components as usual

- The (vector) equation of motion then becomes

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$(\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k} = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

- In order for the last equation to hold, the **i**, **j** and **k** components of the equation must hold separately



- We thus have the following three scalar equations of motion

$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$

- **NOTE:** If particle motion is constrained to be in the *x*–*y* plane, then only the first two equations are to be used

SOLVING PROBLEMS INVOLVING THE EQUATIONS OF MOTION

- Applying the above equations (and subsequent analogous sets in other coordinate systems) to analyze and solve problems generally requires the **synthesis** of many of the concepts and techniques we have encountered throughout the course

- This is reflected in the length / complexity of the “Procedure for Analysis” section (p 120–121) which should be **carefully studied**

- Here we will only note a few key points

FREE BODY DIAGRAMS

- Continue to be a crucial component of the solution of most problems
- Will frequently be multiple FBD's for a given problem, corresponding to distinct but coupled "particles"
- Identification of **all** forces acting on a given particle is of paramount importance
- Particle accelerations now appear in equations (as knowns or unknowns) – may want to include in FBDs

EQUATIONS OF MOTION

Frictional forces

- When problem specification dictates, the kinetic frictional equation must be used for a particle (body) moving across a rough surface

- Recall that the magnitude, F_k , of the kinetic frictional force is given by

$$F_k = \mu_k N$$

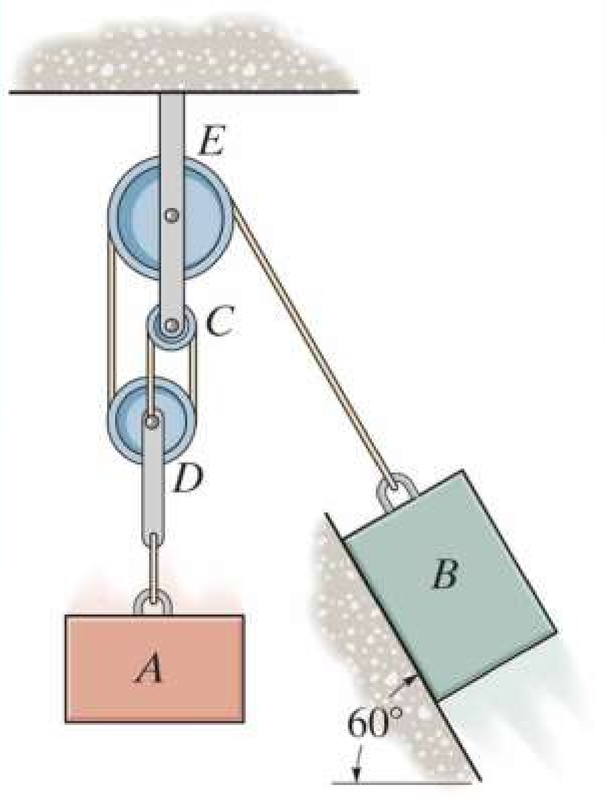
where μ_k is the coefficient of kinetic friction, and N is the normal force

PROBLEM SOLVING: ABOVE ALL, PRACTICE, PRACTICE PRACTICE!!

Problem 13-27 (page 126, 12th edition)

The mass of block *A* is 100 kg. The mass of block *B* is 60 kg. The coefficient of kinetic friction between block *B* and the inclined plane is 0.4. *A* and *B* are released from rest.

(1) Determine the acceleration of block *A* and the tension in the cord. Neglect the mass of the pulleys and the cord.



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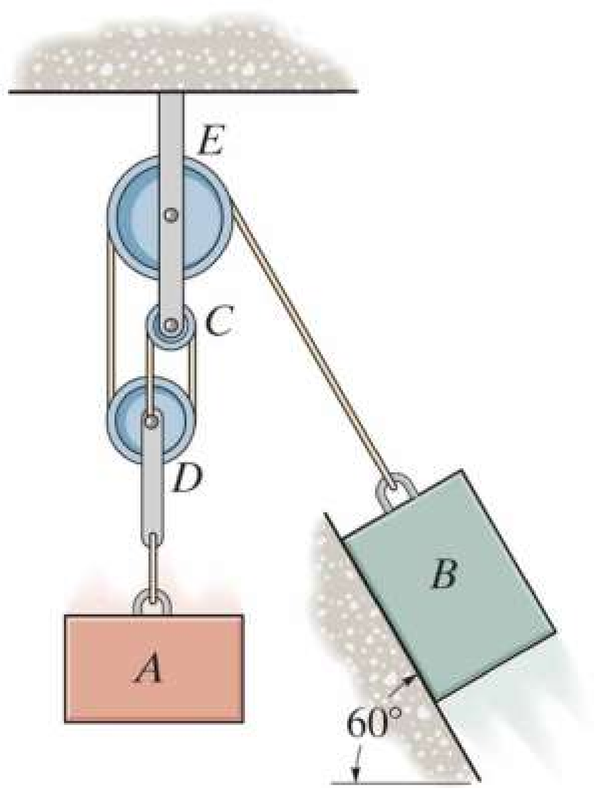

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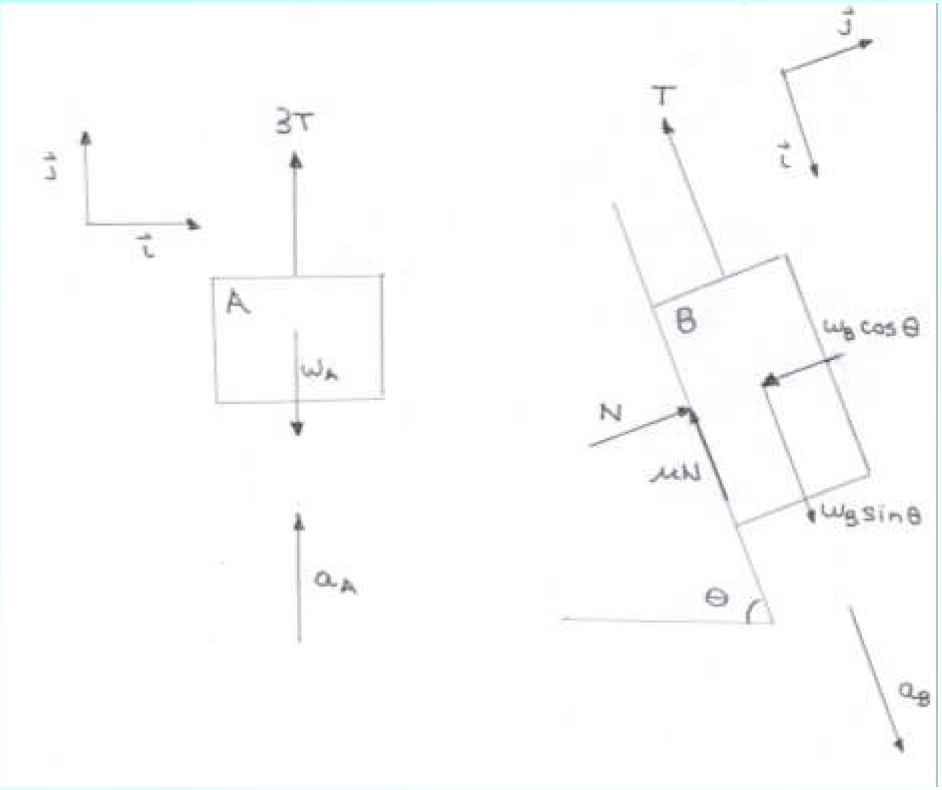
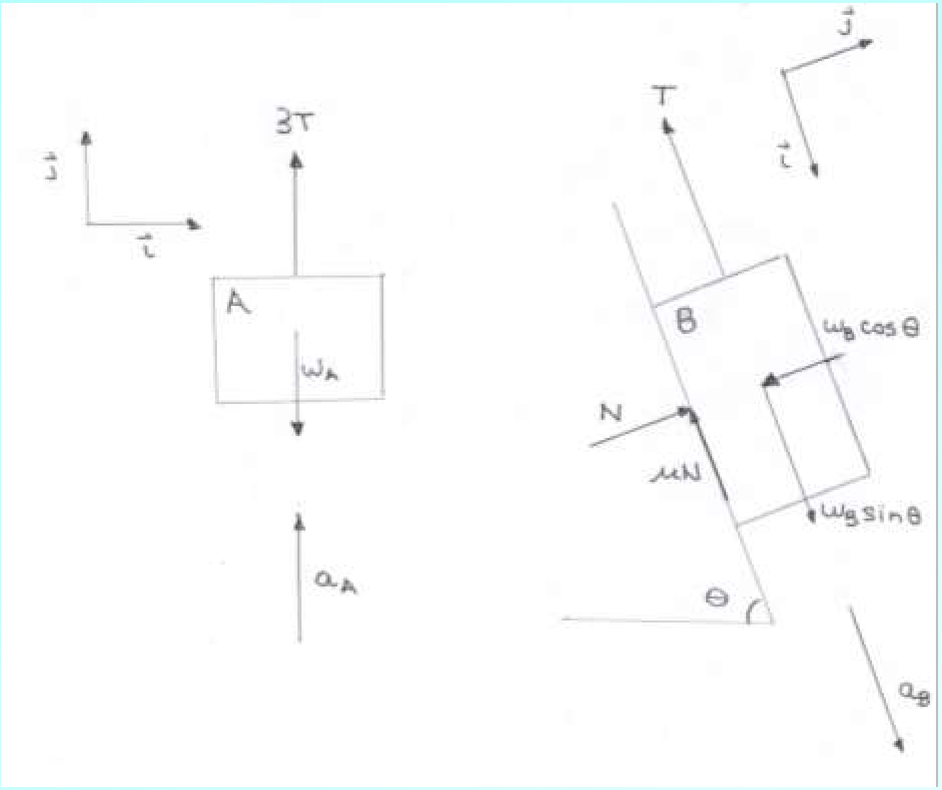


Solution strategy



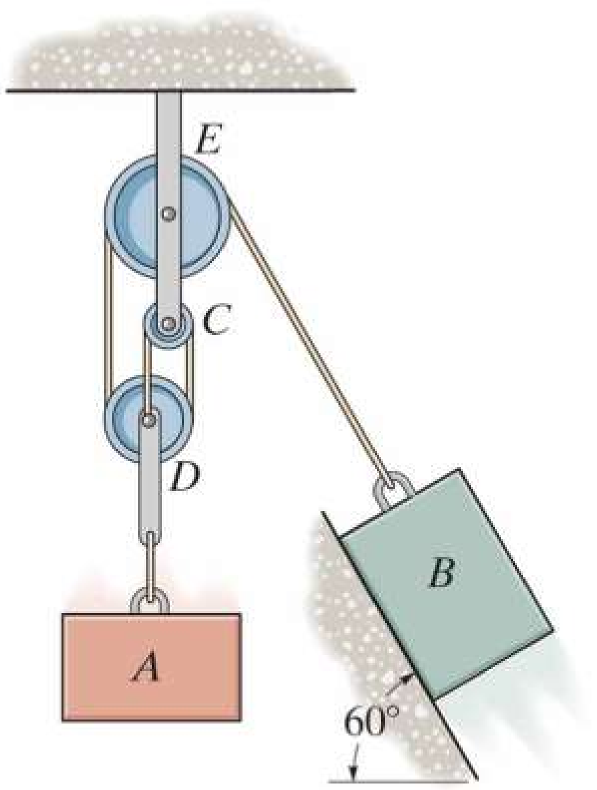
Using free body diagrams, write down equations of motion for two blocks

Relate accelerations of two blocks using dependent motion

Solve equations for a_A and T

Note, we assume that a_A is up and a_B is down. Will need to verify this assumption as part of the solution.

Dependent Motion



PROB13_027.jpg

Exercise:

Show that $a_B = 3a_A$

Assume block A accelerates up. Equation of motion for block A

$$\sum F_y = ma_y : \qquad 3T - m_A g = m_A a_A \qquad (1)$$

If A acclerates up, B accelerates down. Equations of motion for block B. Note that there is no acceleration in the y direction.

$$\sum F_x = ma_x : \qquad m_B g \sin\theta - T - \mu N = m_B a_B \qquad (2)$$

$$\sum F_y = ma_y : \qquad N - m_B g \cos\theta = 0 \qquad (3)$$

Dependent motion

$$a_B = 3a_A \qquad (4)$$

Eliminate T, N and a_B and solve for a_A

$$T = \frac{m_A(a_A + g)}{3} \qquad \text{(from (1))}$$

$$N = m_B g \cos\theta \qquad \text{(from (3))}$$

$$a_B = 3a_A \qquad (4)$$

Substitute these three results in (2), multiply by 3, and rearrange

$$[3m_B(\sin\theta - \mu\cos\theta) - m_A]g = (m_A + 9m_B)a_A$$

$$a_A = \frac{[3m_B(\sin\theta - \mu\cos\theta) - m_A]g}{m_A + 9m_B} = 305 \text{ mm/s}^2$$

and

$$T = \frac{m_A(a_A + g)}{3} = 337 \text{ N}$$

Note that our assumption about the directions of the accelerations of the blocks was correct. If it was not (i.e. if a_A had been computed to be negative), we would need to rework the problem, since our assumption about the direction of the frictional force on block B would also be incorrect.