PHYS 170 Section 101

Lecture 21

October 26, 2018

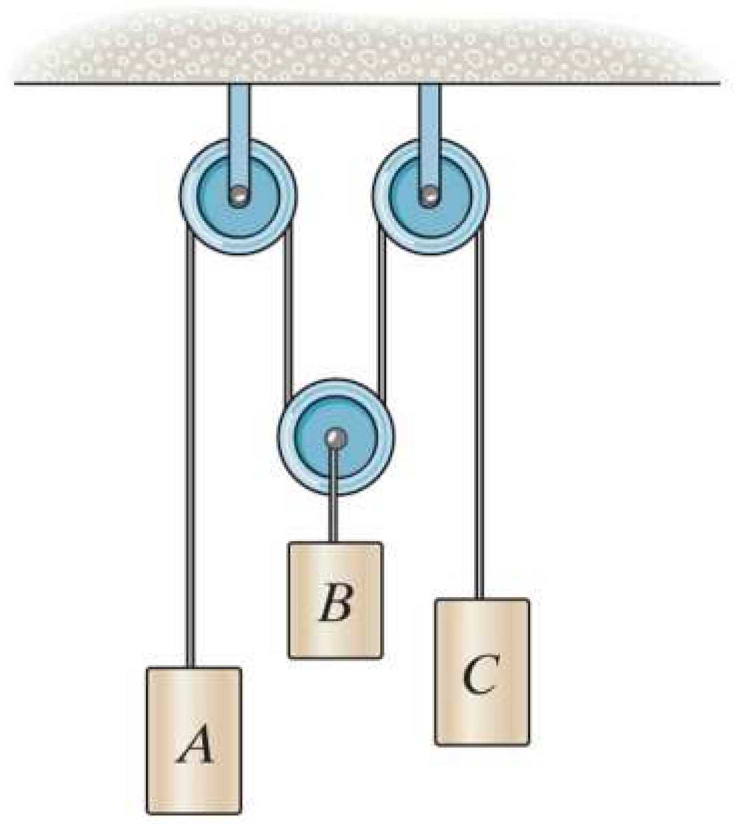
Lecture Outline/Learning Goals

- Worked examples of absolute dependent motion (pulley and block systems)
- 12.10 Relative Motion of Two Particles Using Translating Axes
- Worked example of relative motion

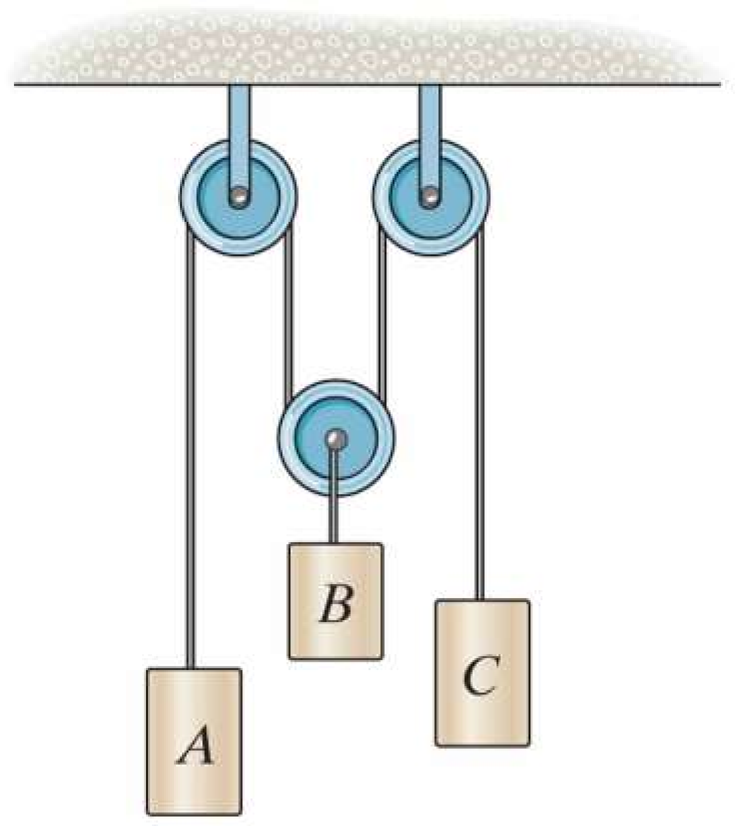
Problem 12-207 (page 95, 13th edition)

Block *A* is moving down at 4 ft/s. Block *C* is moving up at 2 ft/s.

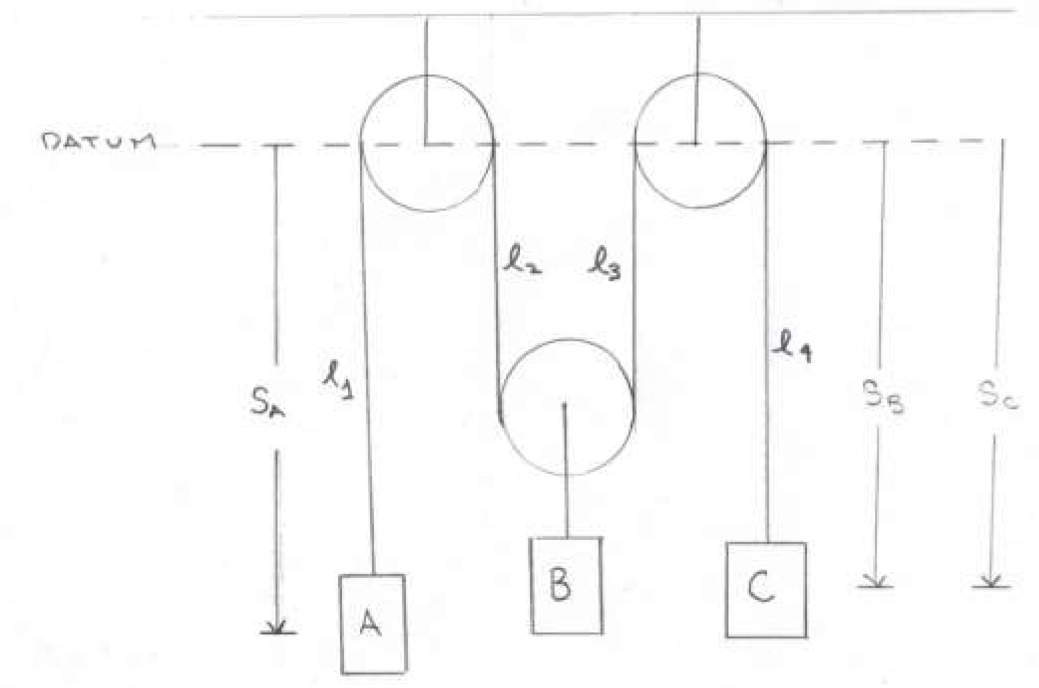
(1) Determine the speed and direction of motion of block *B*.

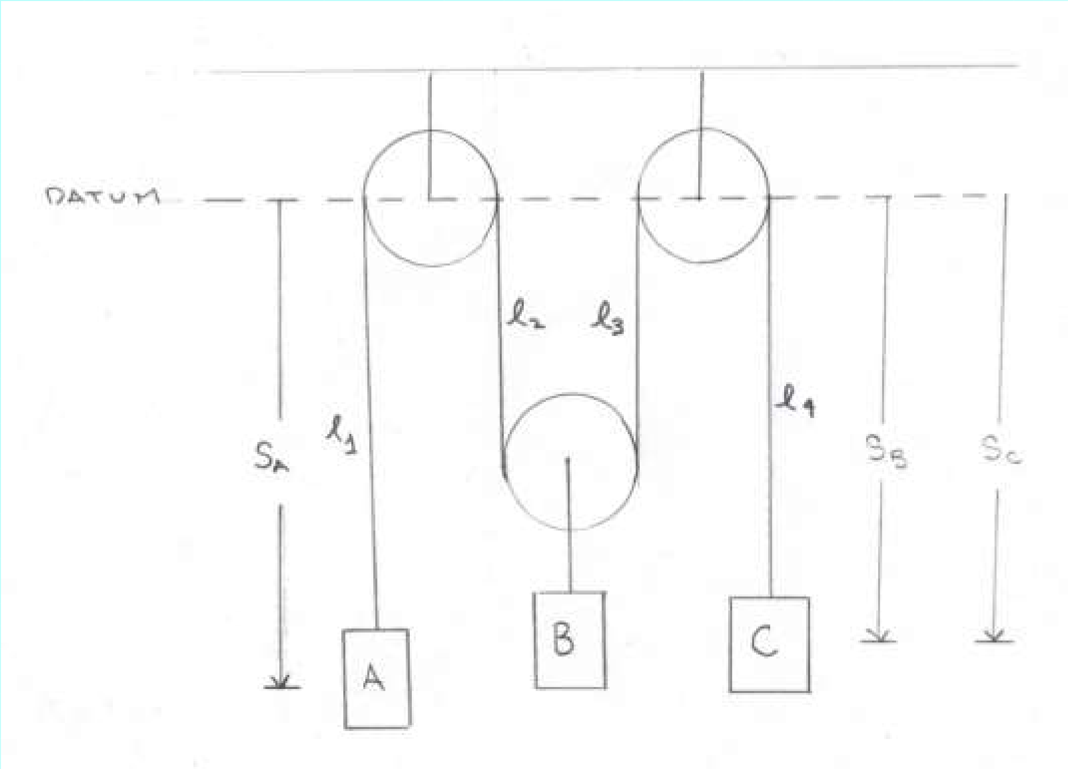


PROB12_207.jpg



PROB12_207.jpg





Solution strategy

Write down rope equation

Write down path equations such that l_i in their sum can be eliminated with the rope eqn.

Determine position equation

Differentiate position equation

Determine speed/direction of block B from given data

Rope equation

$$l_1 + l_2 + l_3 + l_4 = \text{ constant}$$

Path equations (down is positive)

$$l_1 + \text{constant} = s_A$$
$$l_2 + \text{constant} = s_B$$
$$l_3 + \text{constant} = s_B$$
$$l_4 + \text{constant} = s_C$$

Eliminate the l's. Add the four path equations and use the rope equation

$$l_1 + l_2 + l_3 + l_4 + \text{constant} = s_A + s_B + s_B + s_C$$

$$s_A + 2s_B + s_C = \text{constant}$$

$$s_A + 2s_B + s_C = \text{constant}$$

Differentiate with respect to time and solve for v_B

$$v_A + 2v_B + v_C = 0$$

$$v_B = -\frac{1}{2}\left(v_A + v_C\right)$$

With $v_A = 4$ and $v_C = -2$ we have

$$v_B = -\frac{1}{2}(4-2) = -1 \text{ ft/s}$$

So the speed of block *B* is 1 ft/s, and the direction of its motion is up.

Problem 12-212 (page 95, 13th edition)

The cylinder is lifted by a motor and pulley system. The motor draws in cable at 30 cm/s.

(1) Determine the speed of the cylinder.

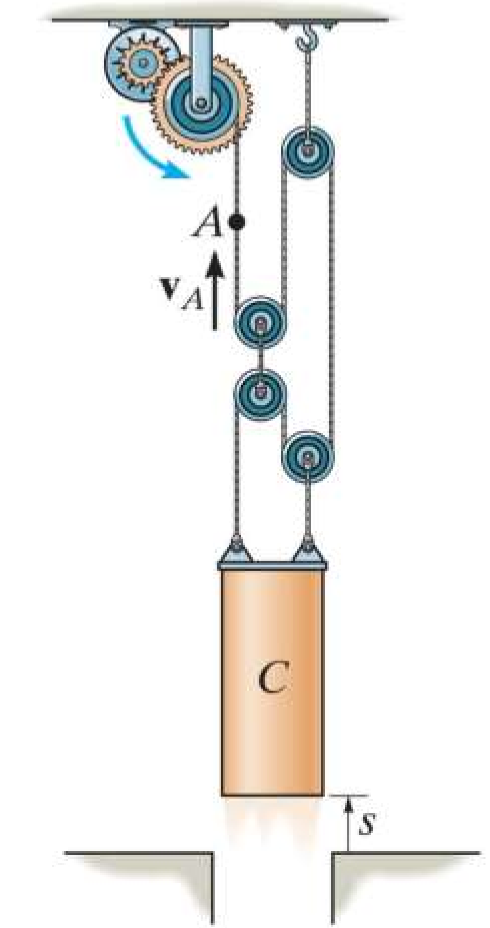


Figure: 12_P212

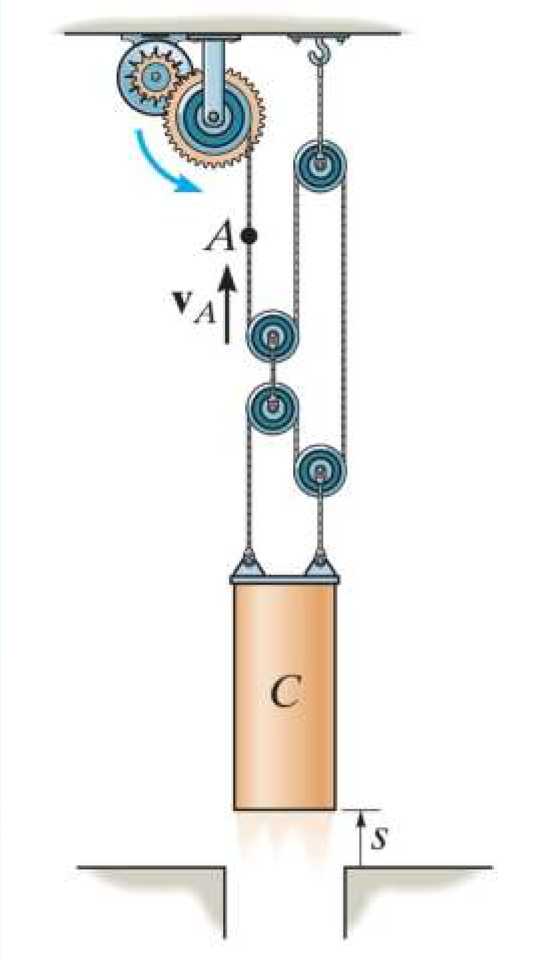
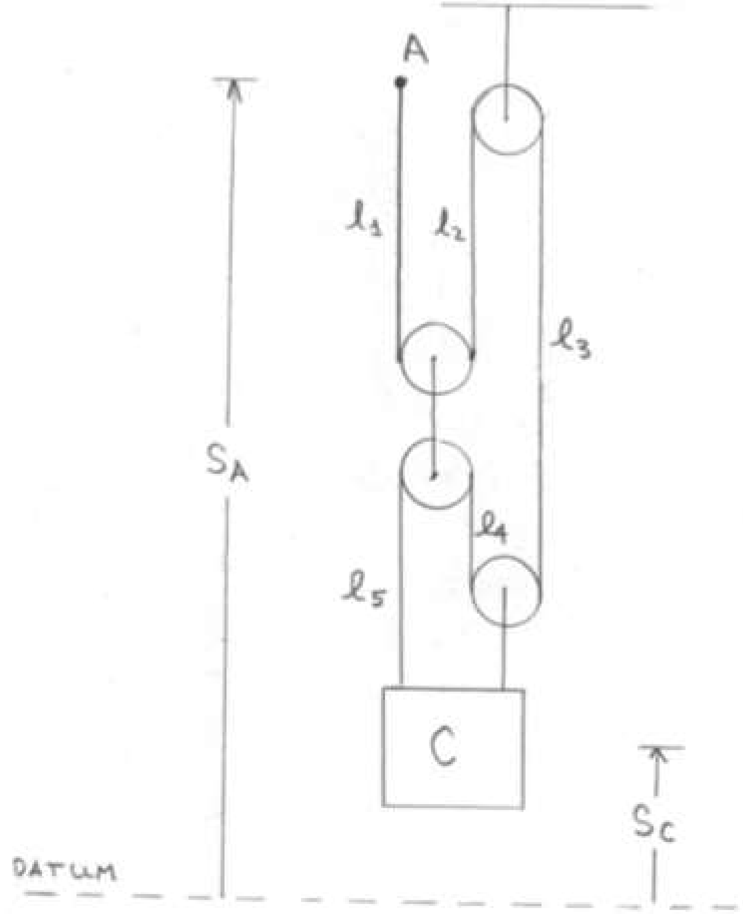
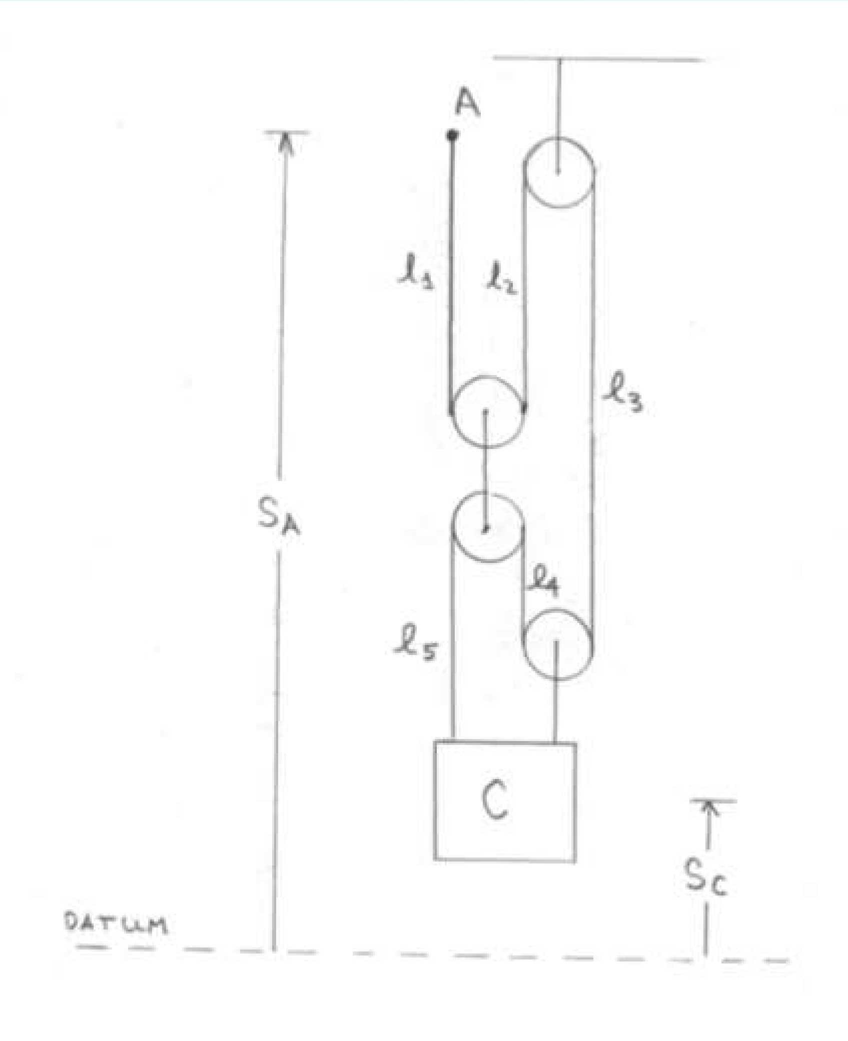


Figure: 12_P212





Solution strategy

Replace the motor by point A which moves up at 30 cm/s

Write down rope equation

Write down path equations such that l_i's in their sum can be eliminated with the rope eqn.

Determine position equation

Differentiate position equation

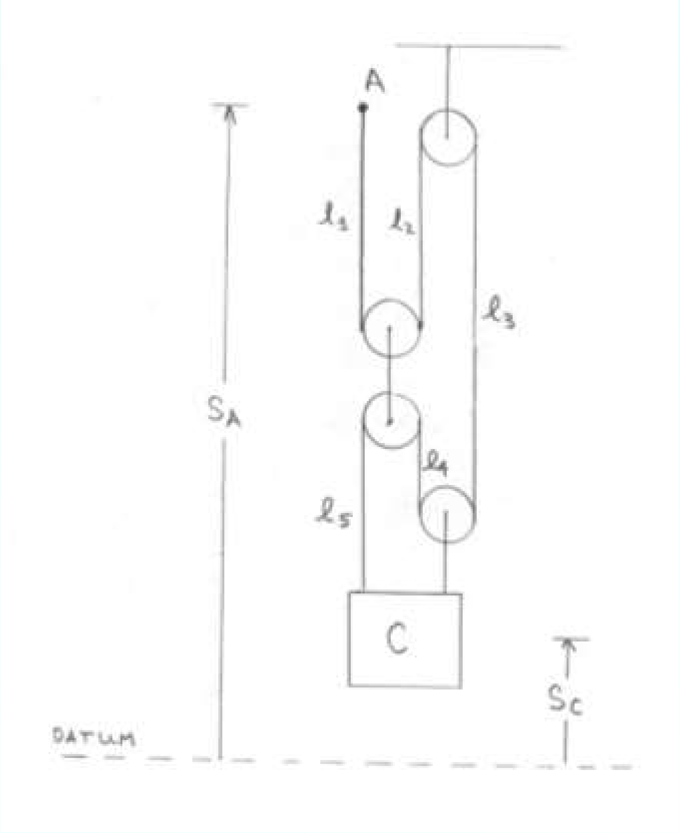
Determine speed of block from given data

Motor is replaced by point A which moves up at 30 cm/s

Rope equation

$$l_1 + l_2 + l_3 + l_4 + l_5 = \text{constant}$$

Datum: Note that datum is drawn below cylinder, and that up is positive



Path equations

$$s_C + l_1 + l_5 + \text{constant} = s_A$$
$$s_C + l_2 + l_4 = \text{constant}$$
$$s_C + l_3 = \text{constant}$$

Add the path equations and use the rope equation

$$3s_C = s_A + \text{constant}$$

Differentiate the last equation (position equation) with respect to time

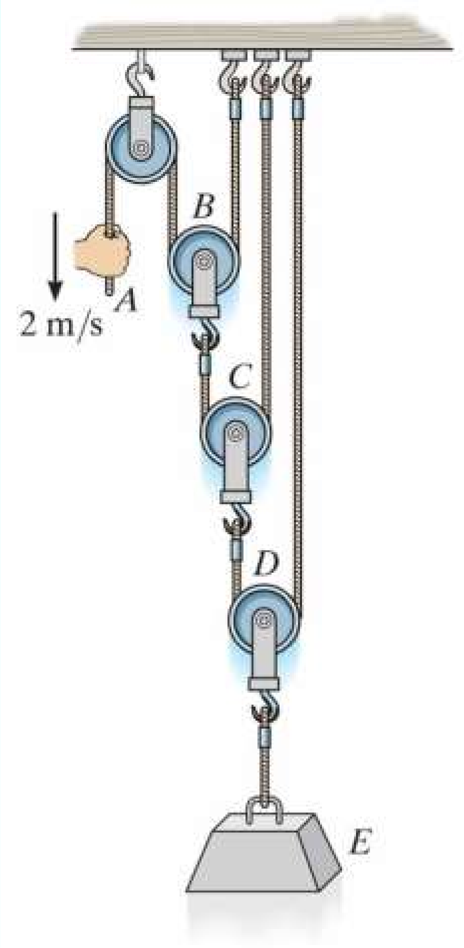
$3v_C = v_A$

Given $v_A = 30.0$ cm/s, we have $v_C = 10.0$ cm/s

Problem 12-208 (page 96, 12^{th} edition)

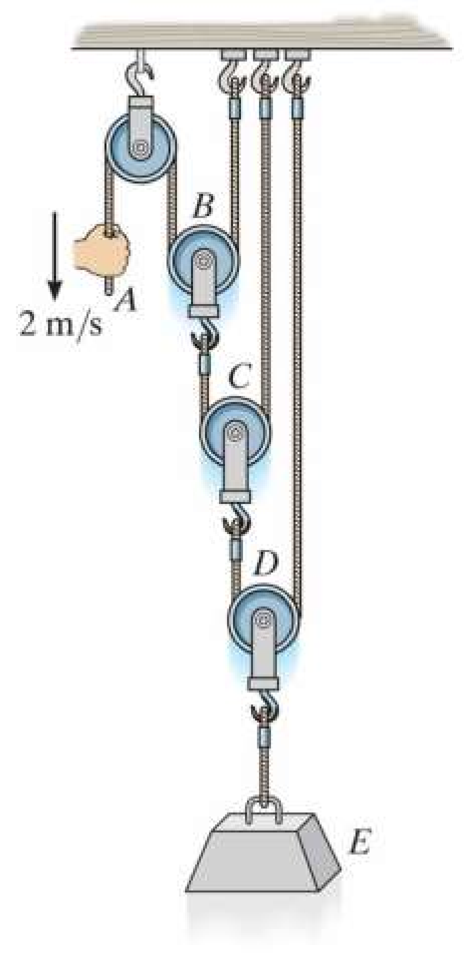
The end of the cable at *A* is pulled down with speed 2 m/s.

(1) Determine the speed at which block *E* rises.

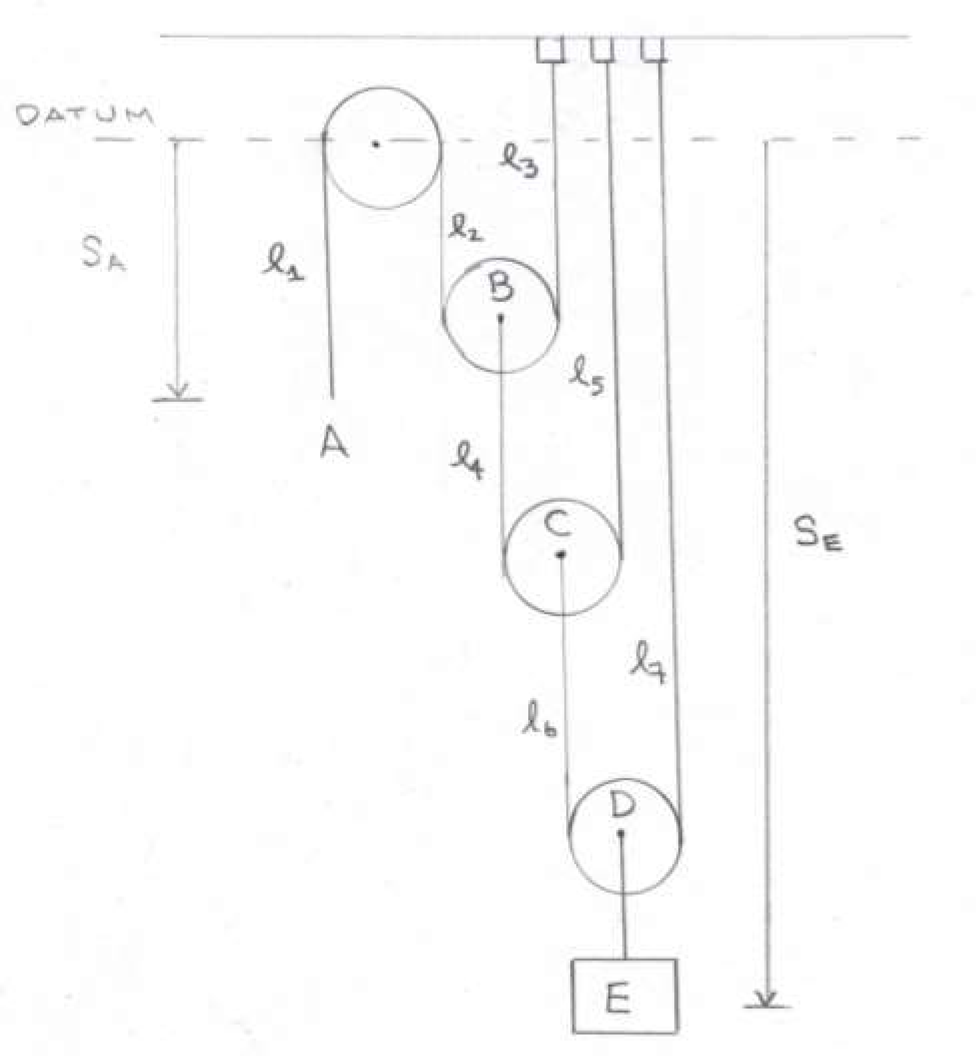


PROB12_208.jpg

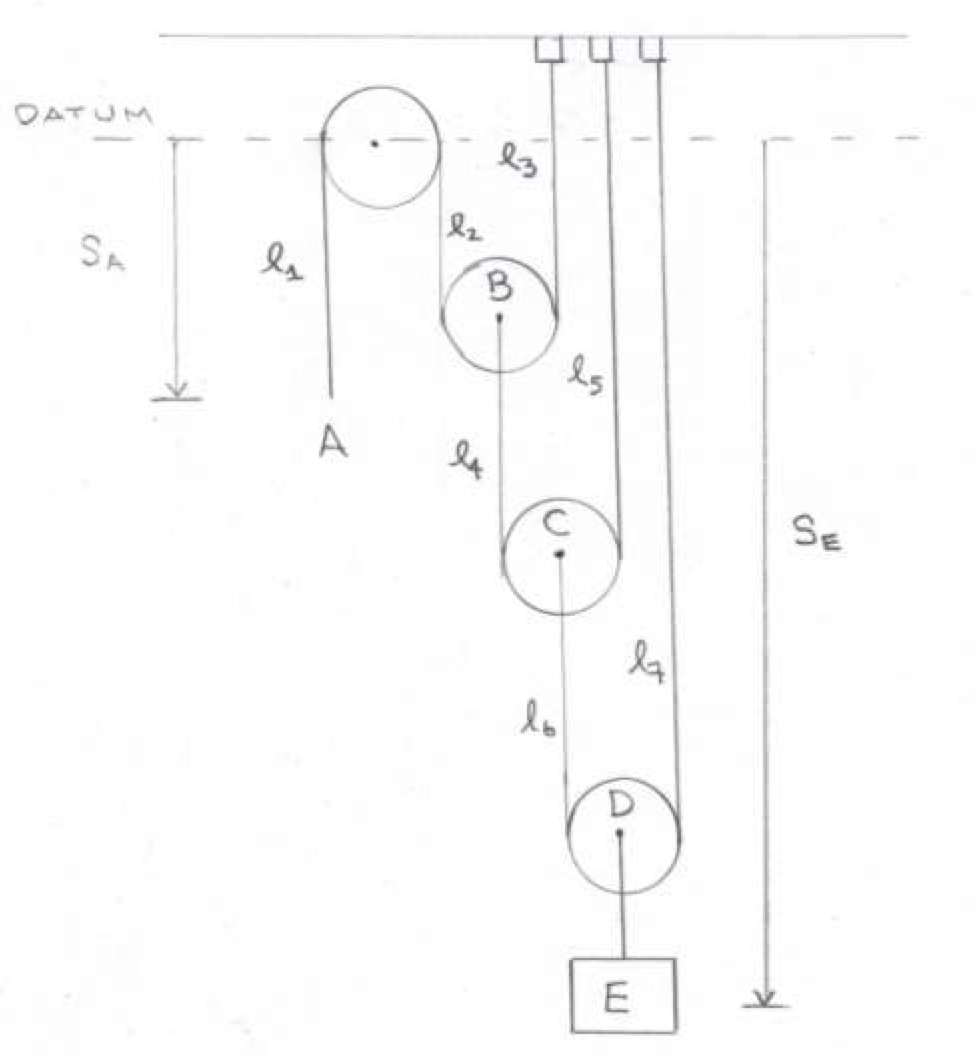
SELF STUDY



PROB12_208.jpg



SELF STUDY



Solution strategy

Write down rope equations (3 of them)

Write down sufficient path equations so that all the l_i appear in at least one equation

Eliminate the l_i's by manipulating the path equations and using the rope equations, until an equation involving only s_A and s_E remains

Differentiate that equation, and use it with the given data to determine the speed of E

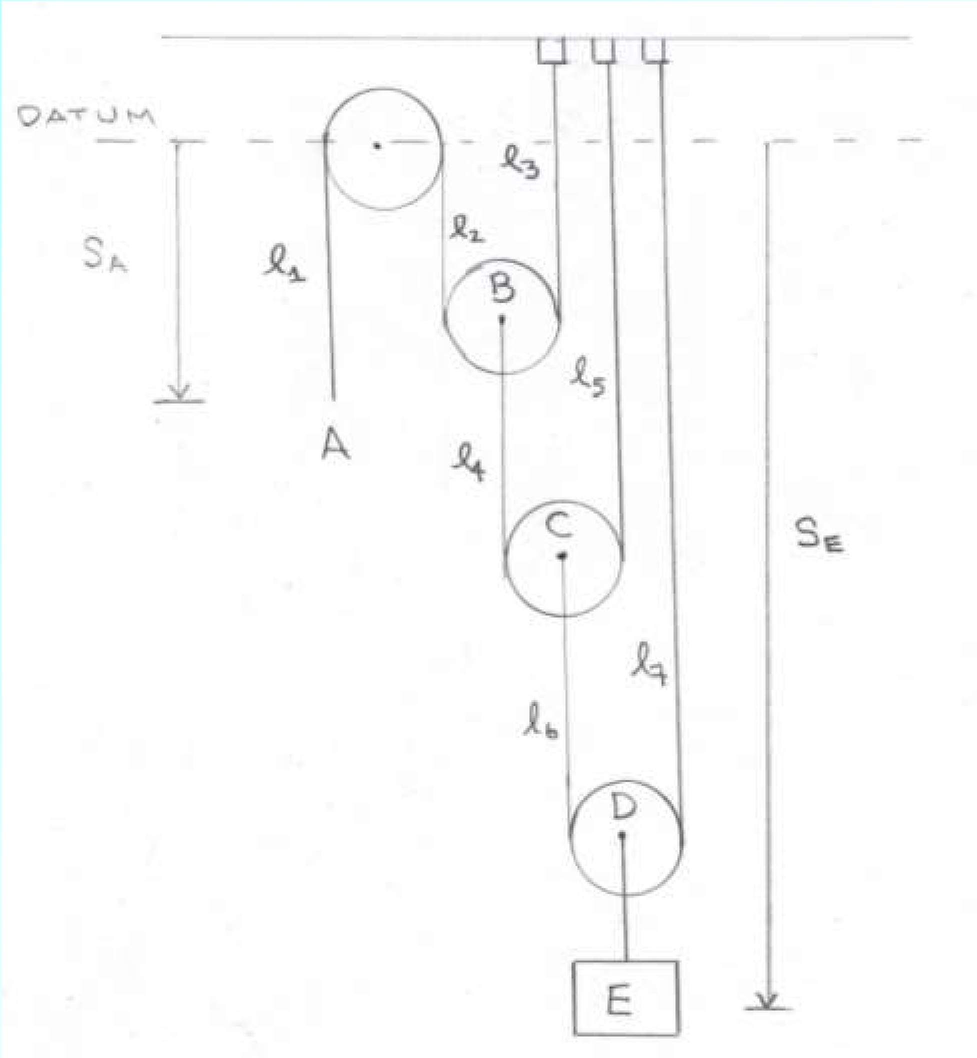
SELF STUDY

Rope equations

$$l_1 + l_2 + l_3 = \text{constant}$$
$$l_4 + l_5 = \text{constant}$$
$$l_6 + l_7 = \text{constant}$$

Path equations (down is positive)

$$l_1 + \text{constant} = s_A$$
$$l_2 + l_4 + l_6 + \text{constant} = s_E$$
$$l_3 + l_4 + l_6 + \text{constant} = s_E$$
$$l_5 + l_6 + \text{constant} = s_E$$
$$l_7 + \text{constant} = s_E$$



SELF STUDY

Eliminate the l's

Add the first three path equations and use the first rope equation

$$2l_4 + 2l_6 + \text{constant} = s_A + 2s_E$$

Add two times the fourth path equation and use the second rope equation

$$4l_6 + \text{constant} = s_A + 4s_E$$

Add four times the fifth path equation and use the third rope equation

$$s_A + 8s_E = \text{constant}$$

Differentiate the last equation with respect to time

$$v_A + 8v_E = 0$$

SELF STUDY

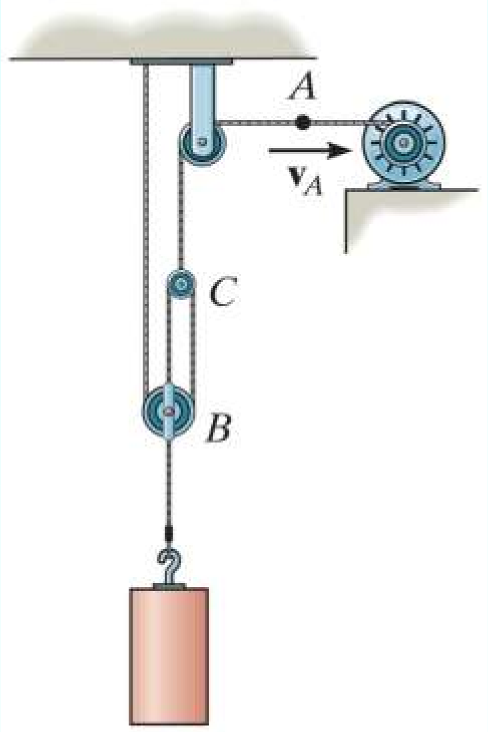
$v_A + 8v_E = 0$

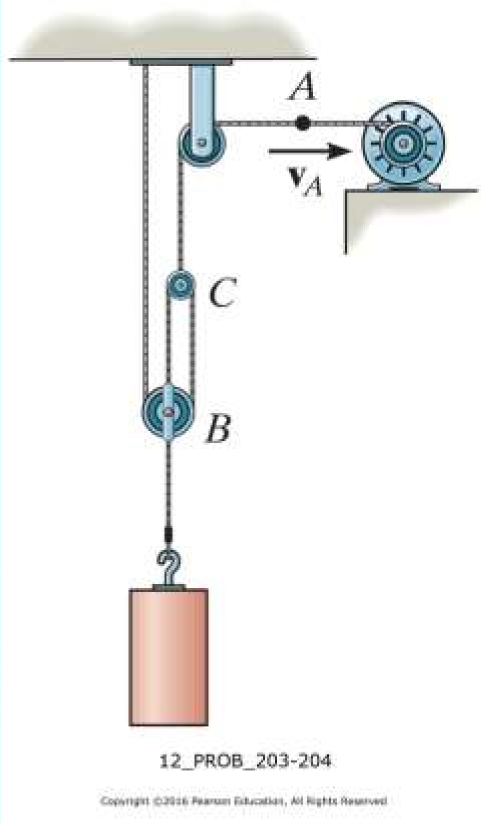
With $v_A = 2$ we have $v_E = -1/4$. Thus, the speed of block *E* is 0.250 m/s and its direction is up.

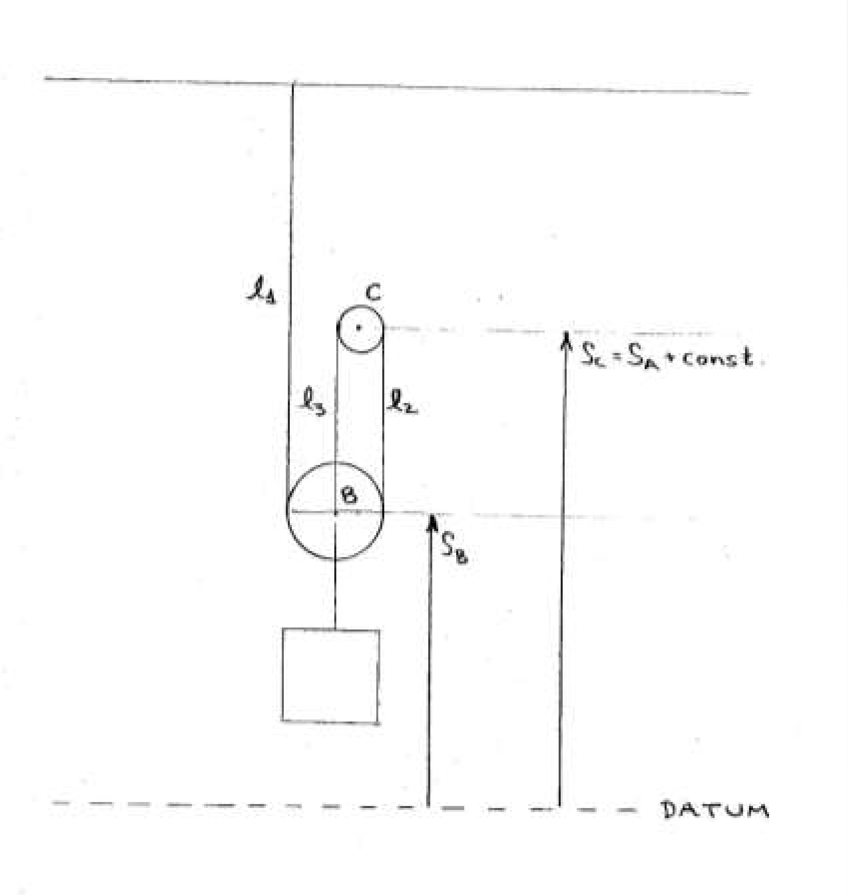
SELF STUDY

Problem 12-204 (page 99, 14th edition)

The cable at A is being drawn toward the motor at $v_A = 8$ m/s. Determine the velocity of the block.







Note: The length of cord between point C and point A is constant.

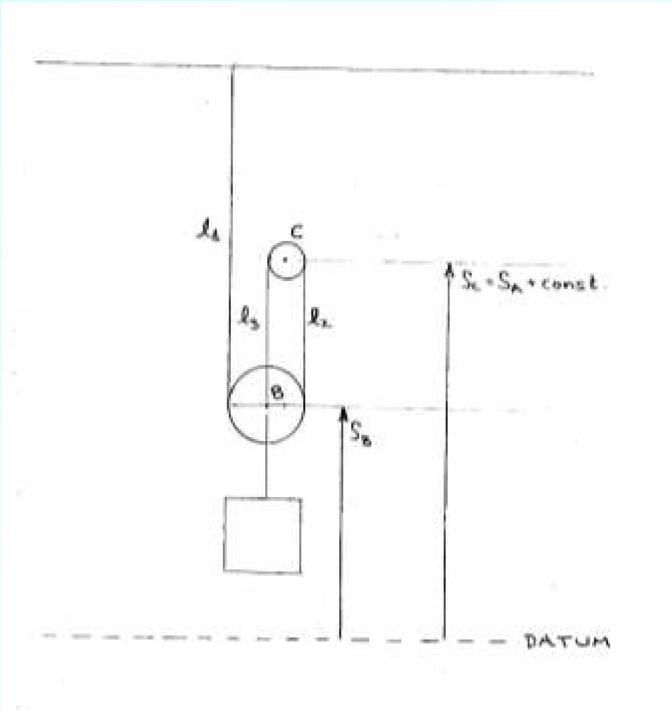
That is, we have:

$$s_C = s_A + \text{constant}$$

Rope equation

$$l_1 + l_2 + l_3 = \text{ constant}$$

Path equations (up is positive)



$$s_B + l_1 = \text{constant}$$

$$s_B + l_2 = s_c = s_A + \text{constant}$$

$$s_B + l_3 = s_c = s_A + \text{constant}$$

Eliminate the *l*'s. Add the three path equations and use the rope equation.

$$3s_B + \text{constant} = 2s_A + \text{constant}$$

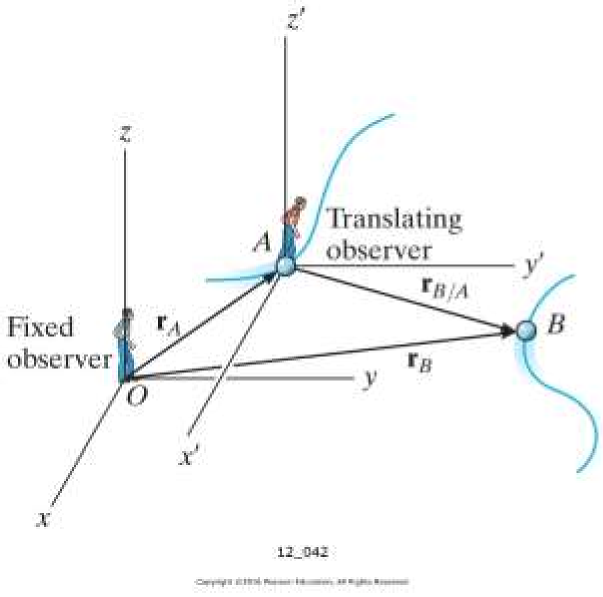
Differentiate with respect to time, then solve for v_B

$$3v_B = 2v_A \;\Rightarrow\; v_B = \frac{2}{3}v_A \;\Rightarrow\; v_B = \frac{2}{3}(8)\text{ m/s} = 5.33\text{ m/s (up)}$$

12.10 Relative-Motion of Two Particles Using Translating Axes

- **MOTIVATION:** In our study of kinematics so far have used single *coordinate system* or *reference frame* with which to describe the motion of particles

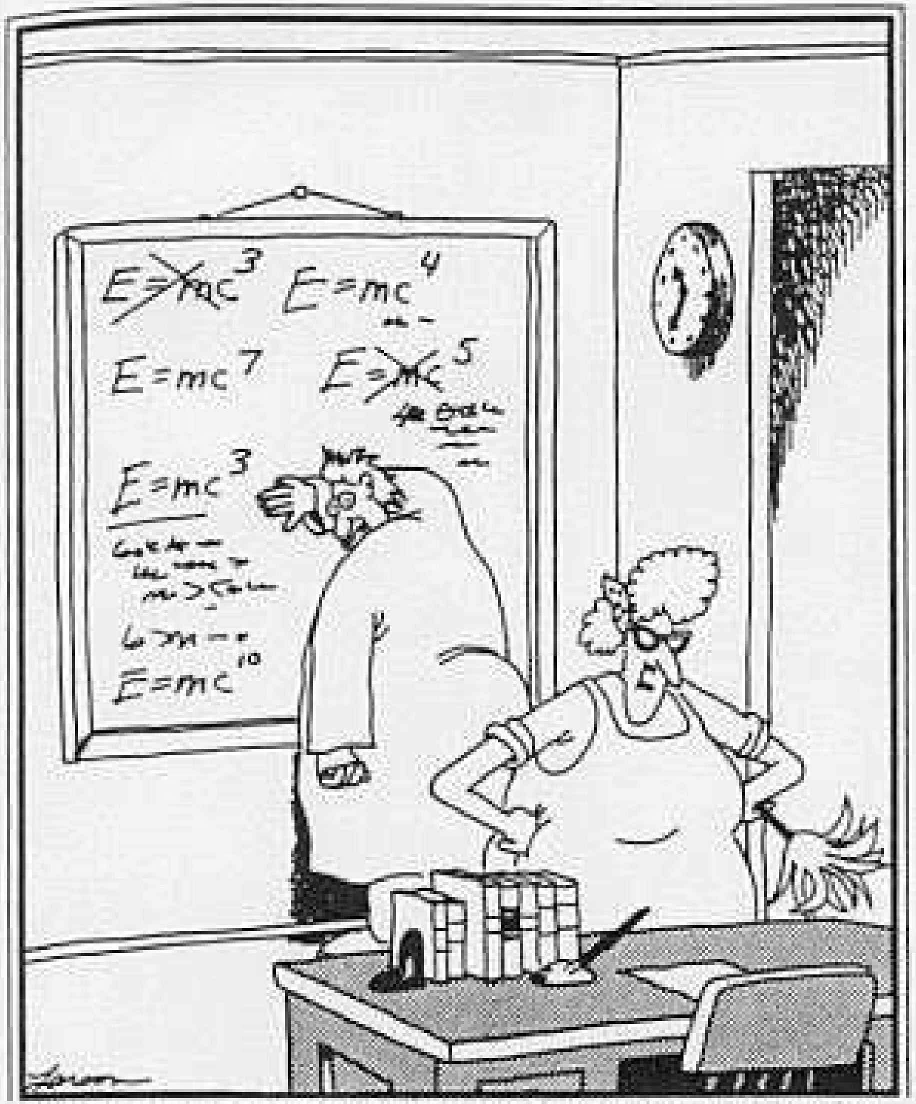
- In some instances, it is more convenient to use two or more reference frames which are *translating* with respect to each other—this may facilitate the description of the overall motion which may be complicated when viewed from a single reference frame



- Consider particles *A* and *B* moving along some arbitrary paths as shown in the figure

- We refer to the *xyz* coordinate system as the fixed reference frame: the *x'y'z'* system, is translating (*but not rotating*) with respect to the fixed frame and its origin tracks the position of observer *A*

Relative-ity

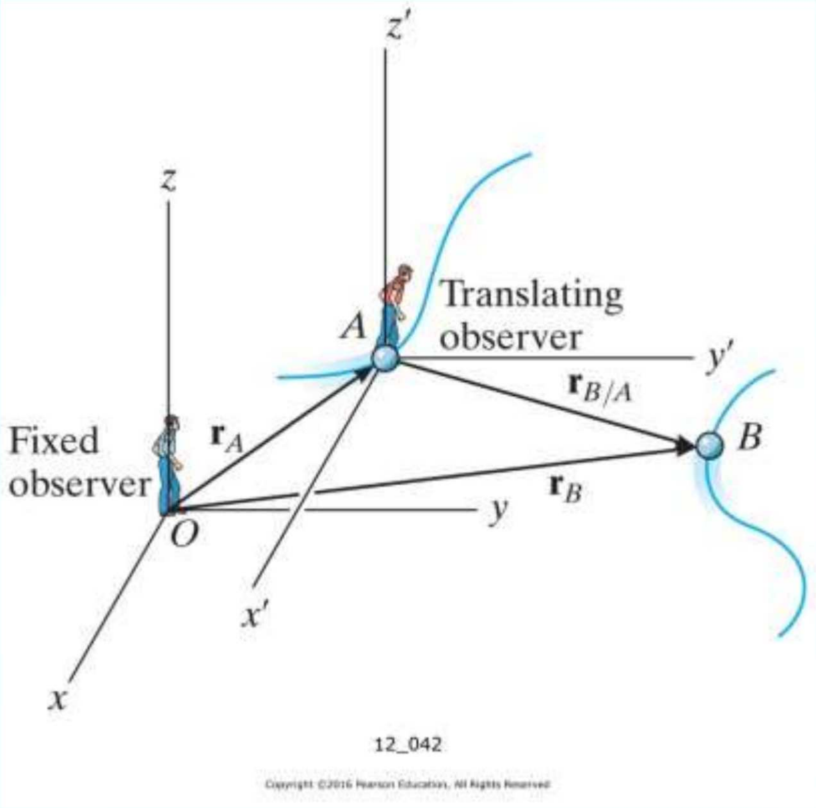


"Now that desk looks better. Everything's squared away, yessir, squaaaaaared away."

- We have the following:

$\vec{r}_A$: Absolute position of observer A (i.e. position with respect to fixed frame)

$\vec{r}_B$: Absolute position of observer B



- The position of B relative to A is given by

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

which we can write as

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

(It is perhaps easiest to remember $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$ and then derive $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$ from it.)

- Taking the time derivative of the last two equations we have

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

- $\vec{v}_A$ and $\vec{v}_B$ are absolute velocities, measured in the fixed reference frame while $\vec{v}_{B/A}$ is the relative velocity of B measured in the translating frame

- Taking the time derivative of the above two equations gives us relations between the absolute and relative accelerations of particles A and B:

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

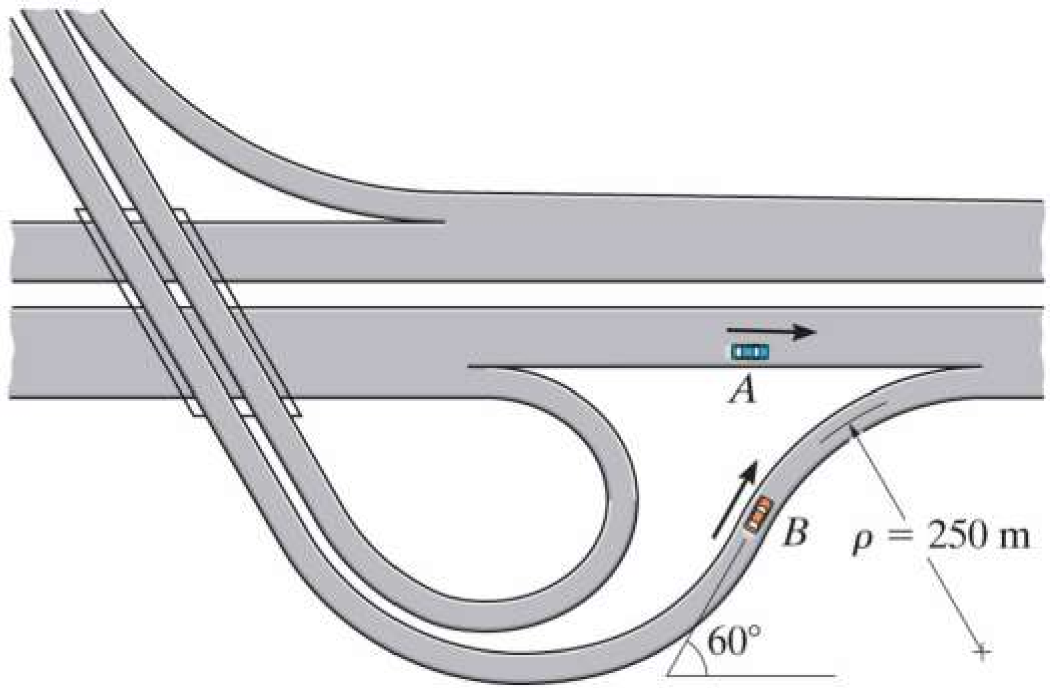
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Problem 12-228 (page 100, 12th edition)

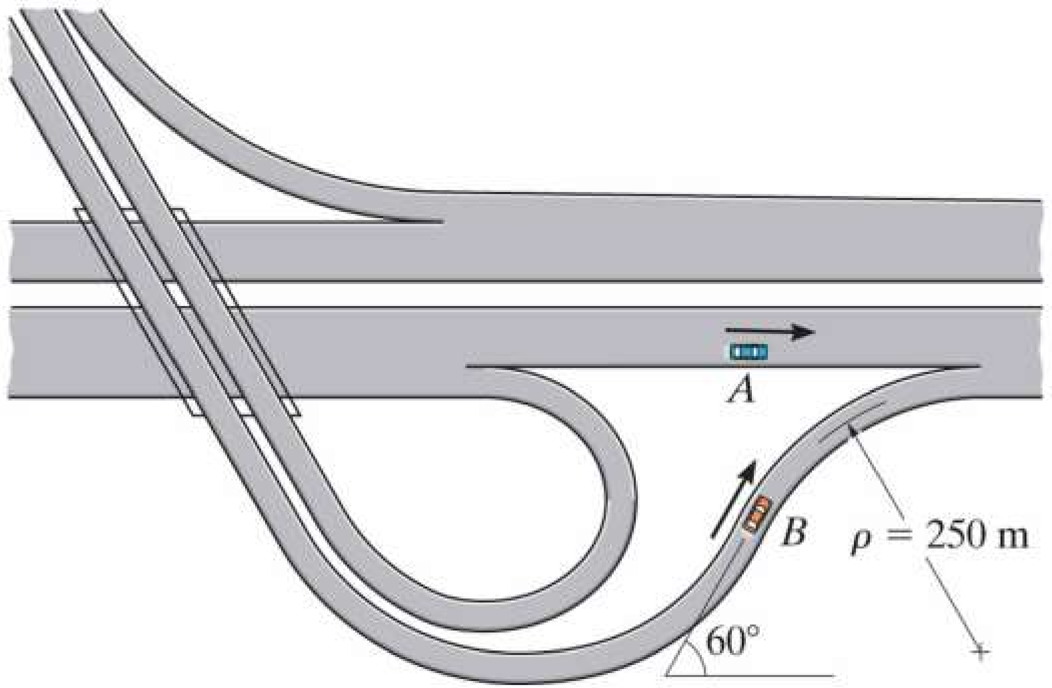
At the instant shown, car *A* travels east along the highway at 30 m/s and accelerates at 2 m/s^2.

At the same instant, car *B* travels on the interchange curve at 15 m/s and decelerates at 0.8 m/s^2.

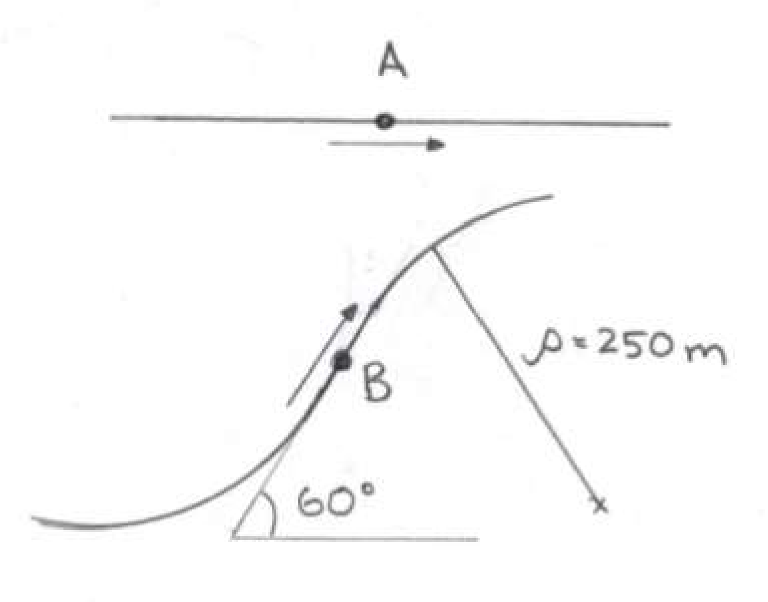
(1) Determine the velocity and acceleration of *B* relative to *A* at this instant.

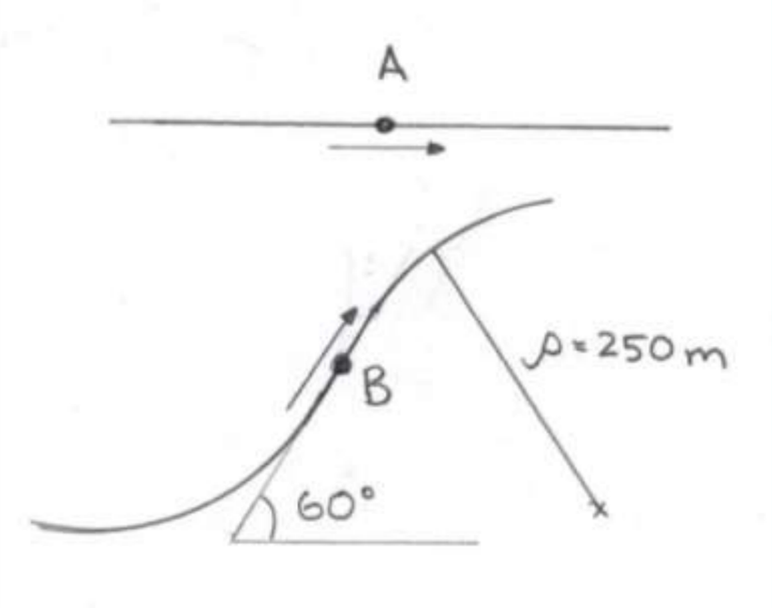


PROB12_228.jpg



PROB12_228.jpg





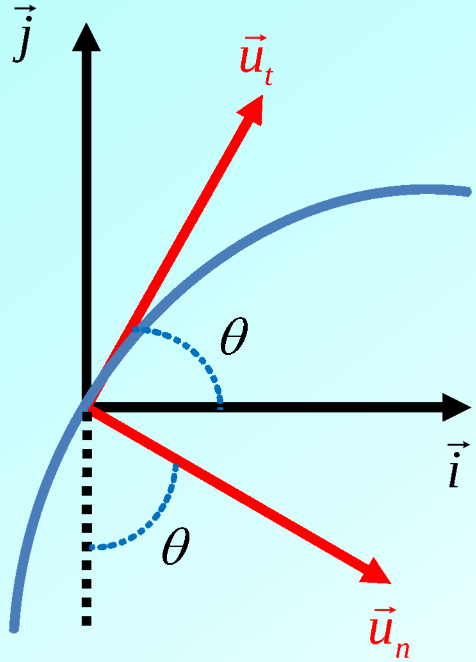
Solution strategy

Express velocities and accelerations in Cartesian components and then use

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$
$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

For car B, express velocity and acceleration in terms of tangential and normal components, and then express $\vec{u}_t$ and $\vec{u}_n$ in terms of $\vec{i}$, $\vec{j}$ and θ

Relationship between tangential/normal and Cartesian unit vectors



$$\vec{u}_t = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$

$$\vec{u}_n = \sin\theta\,\vec{i} - \cos\theta\,\vec{j}$$

Warning!! These are NOT general formulae. They are specific to the particular orientation of the two sets of unit vectors. You need to be able to derive equivalent relationships for other orientations.

Solution continues in Lecture 22