PHYS 170 Section 101
Lecture 17
October 17, 2018

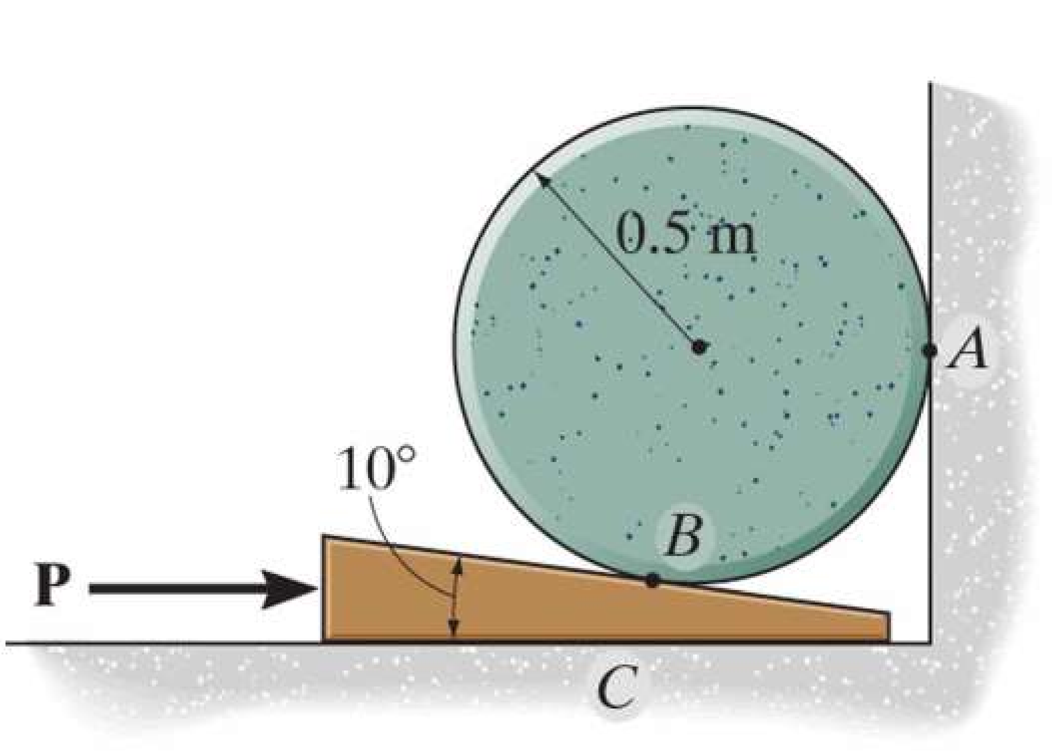
Lecture Outline/Learning Goals

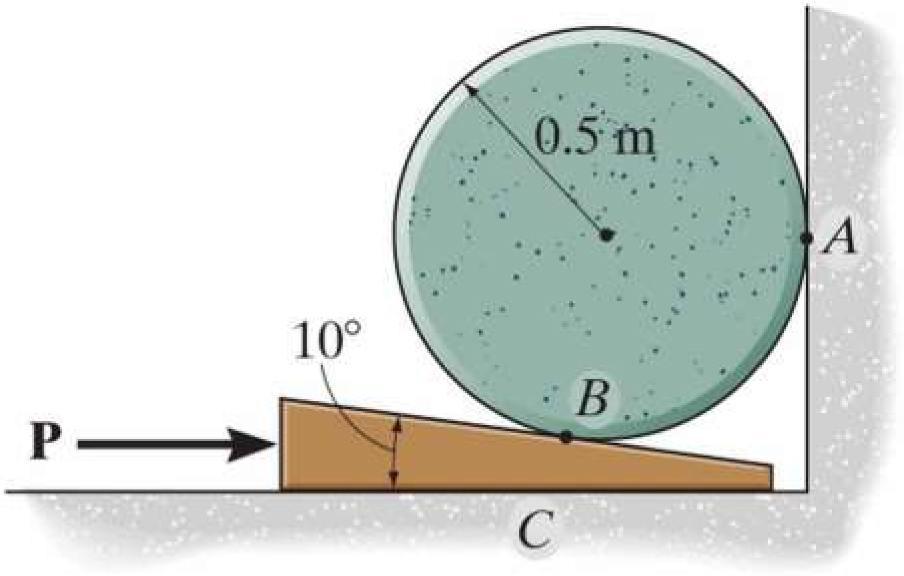
- Wedge problem
- END OF STATICS
- START DYNAMICS
- Rectilinear Kinematics: Continuous Motion (12.2)
- General Curvilinear Motion (12.4)

Problem 8-67 (page 417, 12th edition)

(1) Determine the smallest horizontal force P required to lift the 100 kg cylinder.

The coefficients of static friction at A and B and between the wedge and the ground are $\mu_A = 0.6,\ \mu_B = 0.2,\ \mu_C = 0.3.$





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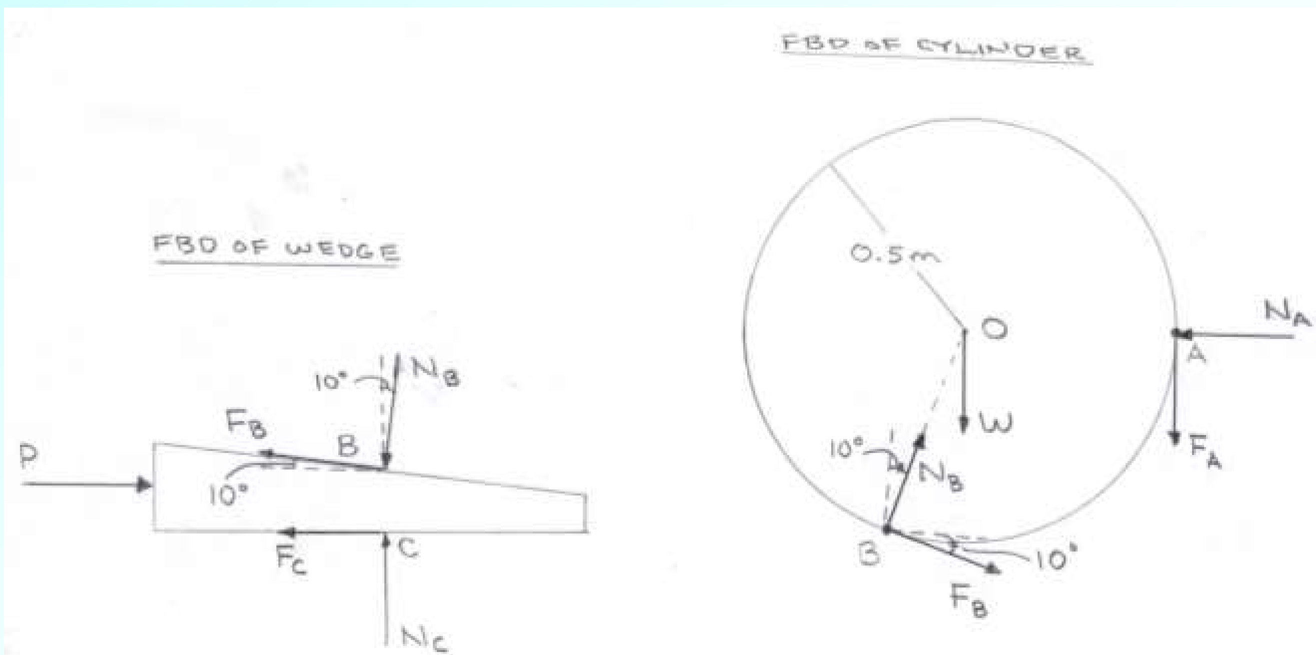
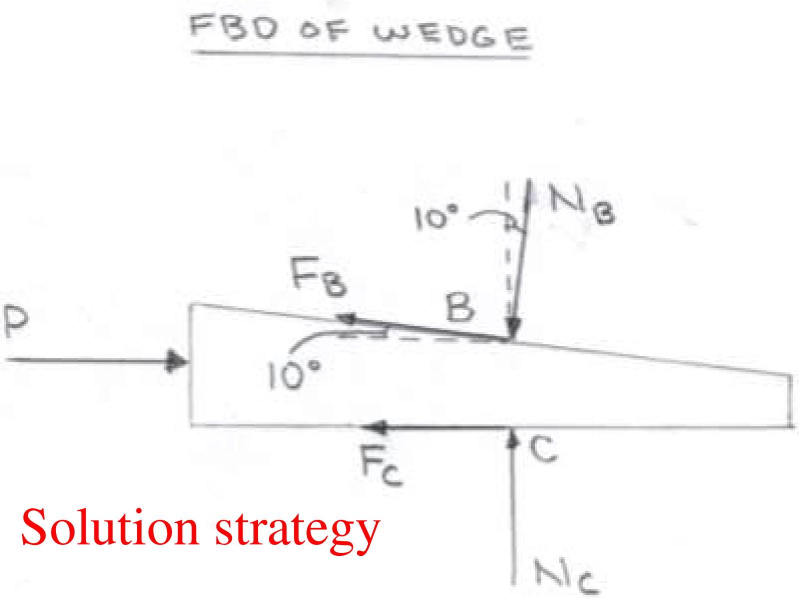
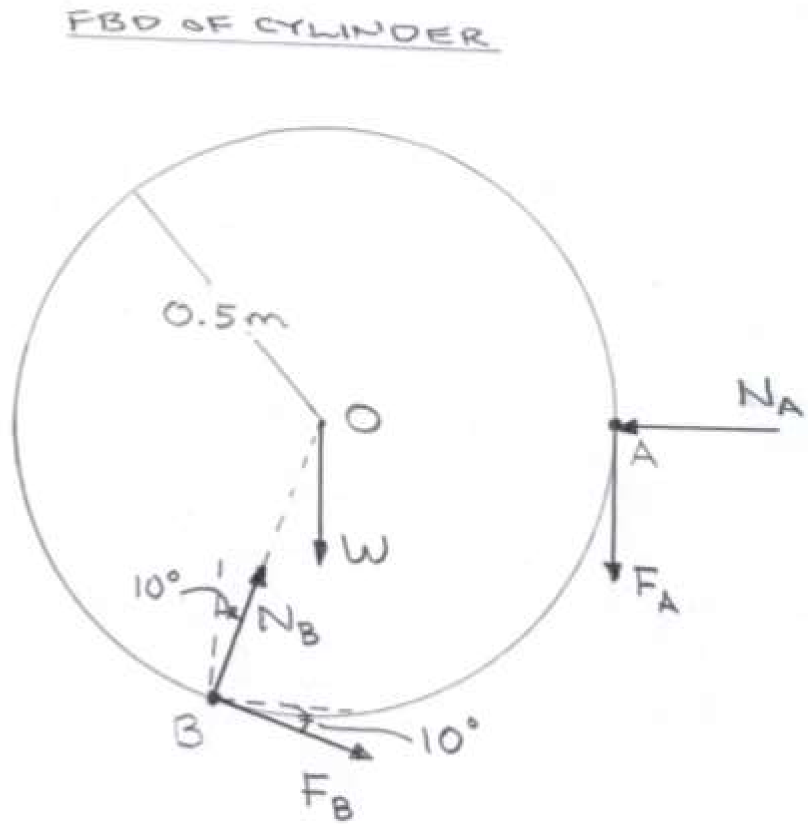






Solution strategy

(1) Formulate Cartesian component equations of equilibrium, determining number of unknowns and equations

(2) Supplement these equations with sufficient additional ones, based on assumption of impending motion at some points, to have a solvable system

(3) Solve, check that remaining restrictions hold, repeat (2)-(3) as necessary

Data

$m = 100 \text{ kg}$ $g = 9.81 \text{ m/s}^2$ $W = mg$ $r = 0.5 \text{ m}$

$\theta = 10^\circ$ $\mu_A = 0.6$ $\mu_B = 0.2$ $\mu_C = 0.3$

Cartesian component equations of equilibrium (Note: no tendency for wedge to rotate, so don't consider moment equilibrium for it)

Wedge:

$$\sum F_x = 0: \quad P - F_C - F_B \cos\theta - N_B \sin\theta = 0 \quad (1)$$

$$\sum F_y = 0: \quad N_C + F_B \sin\theta - N_B \cos\theta = 0 \quad (2)$$

Cylinder:

$$\sum F_x = 0: \quad F_B \cos\theta + N_B \sin\theta - N_A = 0 \quad (3)$$

$$\sum F_y = 0: \quad -F_B \sin\theta + N_B \cos\theta - F_A - W = 0 \quad (4)$$

$$\sum (M_z)_O = 0: \quad F_A r = F_B r \quad (5)$$

Restrictions

$$F_A \le \mu_A N_A \qquad F_B \le \mu_B N_B \qquad F_C \le \mu_C N_C$$

Equations (1) to (5) contain 7 unknowns

$P,\ F_A,\ N_A,\ F_B,\ N_B,\ F_C,\ N_C$

We need two more equations to solve the problem

Assume impending sliding at C and B

$$F_C = \mu_C N_C \qquad (6)$$

$$F_B = \mu_B N_B \qquad (7)$$

Solution to equations (1) to (7)

(Note: Can transform this set to 4 equations in 4 unknowns (N_A, N_B, N_C, P), by making the substitutions $F_B = \mu_B N_B$, $F_C = \mu_C N_C$ and $F_A = F_B = \mu_B N_B$ (from (5)) in equations (1)-(4). Exercise: solve the system)

$P = 857$ N $\quad F_A = F_B = 262$ N $\quad F_C = 373$ N

$N_A = 485$ N $\quad N_B = 1.31$ kN $\quad N_C = 1.24$ kN

Check restrictions (no impending motion or motion at A)

$$F_A < \mu_A N_A = 0.6(485)\text{ N} = 291\text{ N} \Rightarrow$$ assumptions correct

Therefore, the smallest horizontal force required to lift the cylinder is $P = 857$ N

Exercise: Describe what happens to the cylinder, i.e. what is its motion once it starts to move?

Assume impending sliding at C and A

$$F_C = \mu_C N_C \qquad (6)$$

$$F_A = \mu_A N_A \qquad (7)$$

Solution to equations (1) to (7) (Exercise: solve the system)

$P = 1.11$ kN $\qquad F_A = F_B = 364$ N $\qquad F_C = 364$ N

$N_A = 607$ N $\qquad N_B = 1.43$ kN $\qquad N_C = 1.35$ kN

Check restrictions (no motion or impending motion at B)

$F_B > \mu_B N_B = 0.2(1430)\text{ N} = 286\text{ N} \Rightarrow$ assumptions incorrect

END OF STATICS!!

Chapter 12: Kinematics of a Particle

(© Lars Johansson/Fotolia)

12_COC01

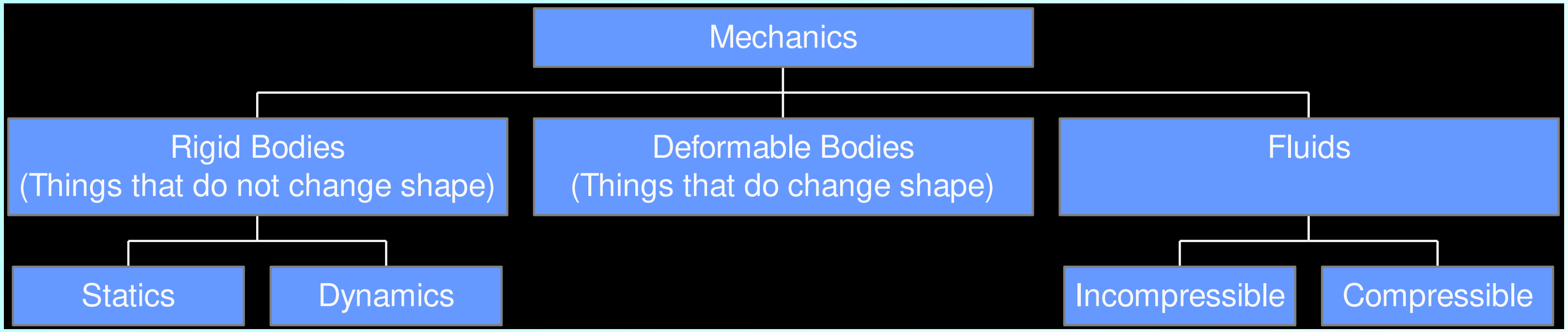
Although each of these boats is rather large, from a distance their motion can be analyzed as if each were a particle.

physics [ˈfɪzɪks]

n (functioning as singular)

1. (Physics / General Physics) the branch of science concerned with using extremely long and complicated formulas to describe how a ball rolls.

Recall from Lecture 1: Branches of Mechanics



- Remainder of course will deal with **dynamics**, i.e. with the motion of bodies that are not in equilibrium, i.e. with **accelerated** motion of bodies
  - Will generally restrict attention to particles, or systems of particles (i.e. size of bodies will not be important and/or considered)

- Dynamics itself will be considered in two parts
  - **Kinematics:** Considers only the geometric aspects of accelerated motion
  - **Kinetics:** Analysis of forces causing accelerated motion, as well as motion per se

Approach and Tools for Remainder of Course

- **PROBLEM SOLVING**
  - Should continue to be **the** key focus of your efforts to master the material
  - Dynamics is generally viewed as more involved than statics due to the need to take into account the motion of bodies in addition to the forces acting on them

- **MATHEMATICAL TOOLS**
  - Many applications will now require **calculus** in addition to vector analysis, algebra and trigonometry
  - Will be working problems in different coordinate systems, e.g. polar [2D] and cylindrical [3D] coordinates in addition to Cartesian coordinates

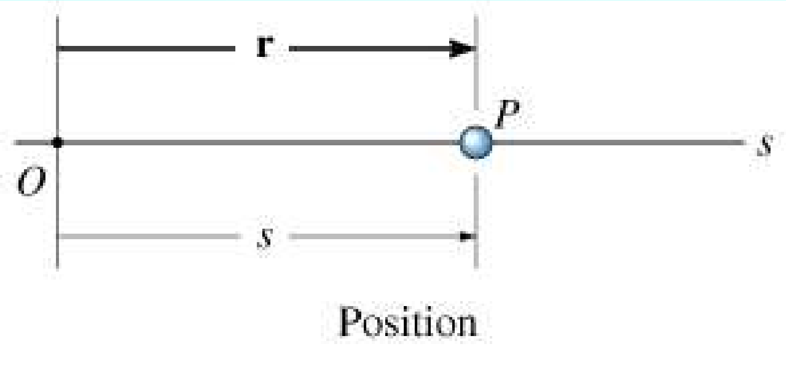
- **BASIC PHYSICS CONCEPTS ARE IMPORTANT AS WELL!!**

12.2 Rectilinear Kinematics: Continuous Motion

- Here we restrict attention to straight line motion of a particle (has mass, but negligible size, shape unimportant)

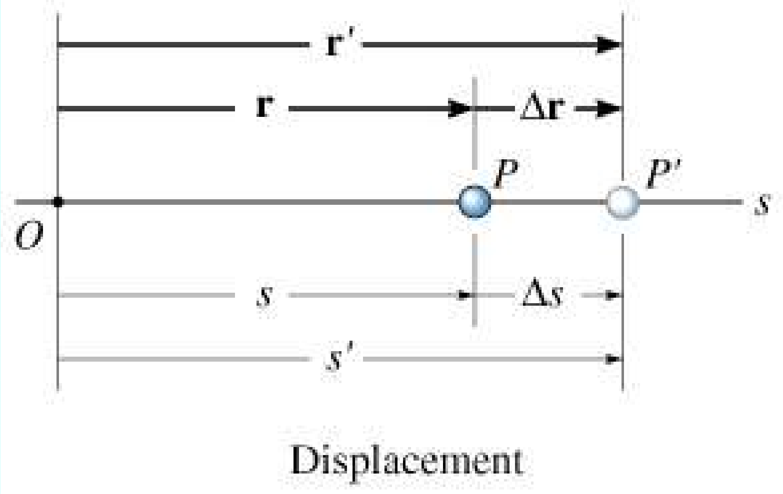
- For bodies with finite size, approximation as a particle requires that center of mass be used for description of motion, and that rotational effects be ignored

- **RECTILINEAR KINEMATICS**: Specified by giving particle's **position**, **velocity** and **acceleration** as a function of time



POSITION

- Use single coordinate axis, *s,* with origin at *O*
- Position vector, **r**, is always along this axis, so can represent the particle position with an algebraic (signed) scalar, *s*
- Typical units: m, ft, …



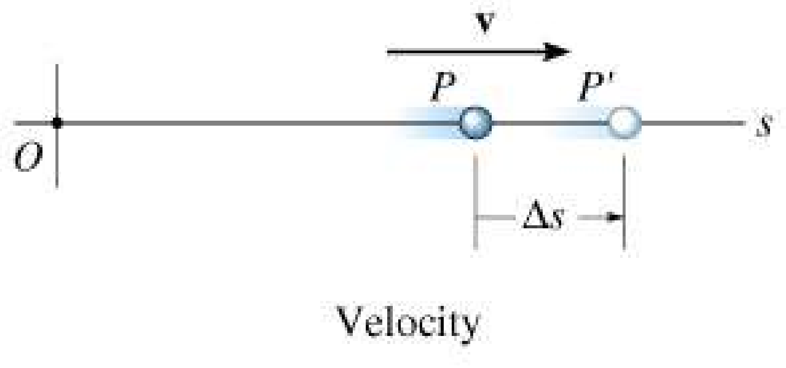
DISPLACEMENT

- Definition: Change in position

Vector: $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$

Scalar: $\Delta s = s' - s$

- Note: Displacement is a vectorial quantity; must be distinguished from distance traveled, which is a positive scalar quantity
- Typical units: m, ft, …



VELOCITY

- Definition: Rate of change of position with respect to time

Average velocity

- If the displacement of the particle is $\Delta\mathbf{r}$ over a time interval Δt, then

$$\mathbf{v}_{avg} = \frac{\Delta\mathbf{r}}{\Delta t}$$

Instantaneous velocity (velocity)

- If the particle displacement is a continuous function of time (assumption in this section), then the instantaneous velocity, $\mathbf{v}$, is defined by

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t}$$

or

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

Instantaneous velocity (velocity) – scalar form

$$v = \frac{ds}{dt}$$

- The sense of v is the same as that of ds (Δs)– the text uses an arrow notation to emphasize the sense; we won't adopt this notation in these notes

Speed

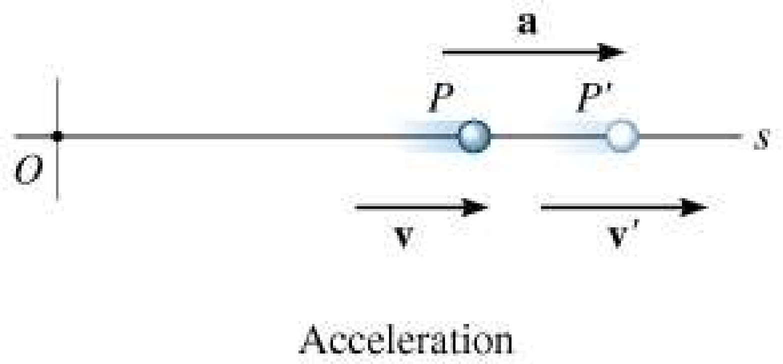
- Definition: Magnitude of velocity (text denotes this by v_{sp} since v as defined above is a signed quantity)

- Typical units for velocity or speed: m/s, ft/s, …

Average speed

- Definition: Positive scalar given by total distance, s_T, traveled in a given time interval Δt

$$(v_{\mathrm{sp}})_{\mathrm{avg}} = \frac{s_T}{\Delta t}$$



ACCELERATION

- Definition: Rate of change of velocity with respect to time

Average acceleration

- If the change in velocity of a particle is $\Delta \mathbf{v}$ over a time interval Δt, then

$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Instantaneous acceleration (acceleration)

- If the particle velocity is a continuous function of time (assumption in this section), then the instantaneous velocity, **a**, is defined by

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

or

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

Instantaneous acceleration (acceleration) – scalar form

$$a = \frac{dv}{dt}$$

- Typical units for acceleration: m/s^2, ft/s^2, …

- Since $v=ds/dt$, we have

$$a = \frac{d^2 s}{dt^2}$$

- **Note:**
  - a is a signed quantity: if $a < 0$, then particle is slowing down or **decelerating**
  - If $a = 0$, then velocity, v, is **constant**

- Now, start from $a = dv/dt$, and multiply both sides by ds (note that treatment of differential quantities, such as ds, dt, dv, … as algebraic values can be rigorously justified)

$$a\,ds = ds\,\frac{dv}{dt} = \frac{ds}{dt}\,dv = v\,dv$$

or

$$a\,ds = v\,dv$$

- This last equation is useful in determining the velocity of a particle when the acceleration is given as a function of position

Constant acceleration

- We now consider the important special case where the acceleration, a, is a **constant**, a_c
- In this case, we can integrate various equations from above to get formulae relating a_c , v, s and t.

Velocity as a function of time

Start from $a_c = \dfrac{dv}{dt}$, so $a_c\, dt = dv$

Integrating over the time interval $[0, t]$, with the velocity at $t = 0$

given by v_0, we have

$$\int_{v_0}^{v} dv = \int_0^t a_c\, dt$$

so

$$v - v_0 = a_c t$$

or

$$v = v_0 + a_c t \qquad \text{constant acceleration}$$

Position as a function of time

Start from $v = \frac{ds}{dt}$, so $v\,dt = ds$

Integrating over the time interval $[0, t]$, with the position at $t = 0$ given by s_0, we have

$$\int_{s_0}^{s} ds = \int_0^t v\,dt = \int_0^t (v_0 + a_c t)\,dt$$

so

$$s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

or

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$ constant acceleration

Velocity as function of position

Start from $v\,dv = a\,ds$ and integrate, using the initial values s_0 and v_0 for the position and velocity, respectively:

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

Thus,

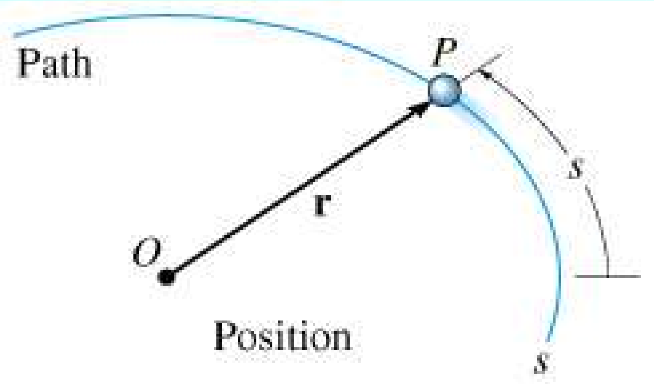
$$\frac{v^2}{2} - \frac{v_0^2}{2} = a_c(s - s_0)$$

or

$$v^2 = v_0^2 + 2a_c(s - s_0) \qquad \text{constant acceleration}$$

- **IMPORTANT:** The above formulae are valid **only** for the case where the acceleration is **constant**, such as for a freely falling body close to the surface of the Earth

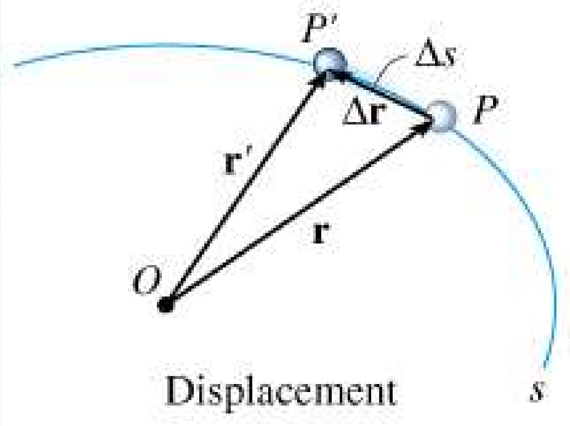
12.4 General Curvilinear Motion



POSITION

- Particle moves along path defined (parameterized) by **path function**, s
- Relative to point O (typically origin of coordinate system), location of particle is given by the position vector, **r**

$$\mathbf{r} = \mathbf{r}(t)$$

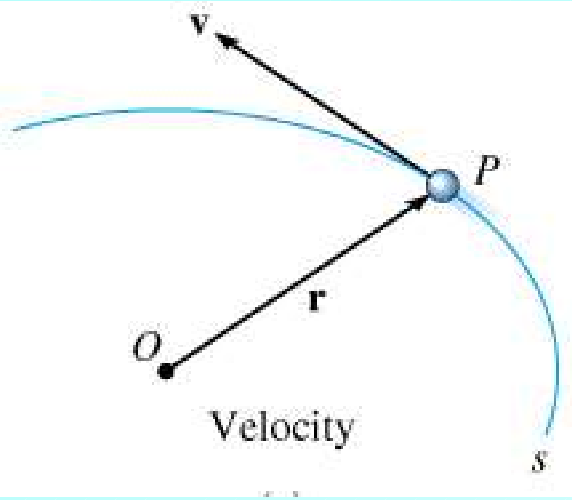


DISPLACEMENT

- Change in position of particle over some time interval Δt , during which the particle traverses a distance Δs along the path

$$\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$$

- Note that the magnitude of the displacement, Δr, can be viewed as a straight-line distance that approximates Δs



VELOCITY

Average velocity

- Over the time interval Δt , the average velocity is given by

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

Instantaneous velocity (velocity)

- Take the limit $\Delta t \to 0$ in the above, so that the direction of $\Delta \mathbf{r}$ approaches the **tangent** to the curve at *P*

- Then the instantaneous velocity is given by

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

or

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

Instantaneous velocity (continued)

- **NOTE:** $d\mathbf{r}$ is tangent to the curve at P, so the **direction** of $\mathbf{v}$ is also tangent to the curve
- The **magnitude** of $\mathbf{v}$, which is again called the **speed**, can be determined by considering the magnitude of $d\mathbf{r}$ in the limit that $\Delta t \to 0$
- In this limit we have

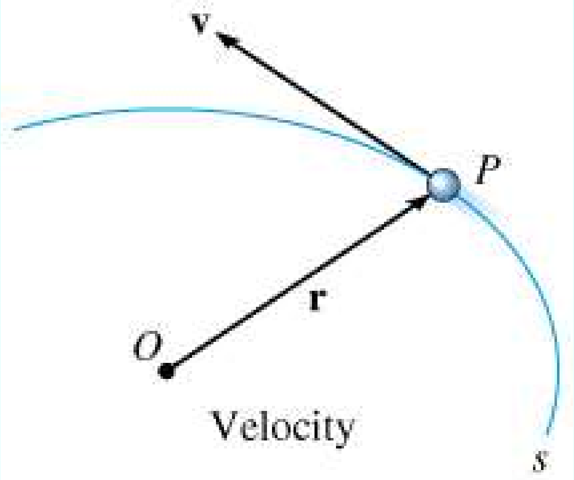
$$\lim_{\Delta t \to 0} \Delta r = \Delta s$$

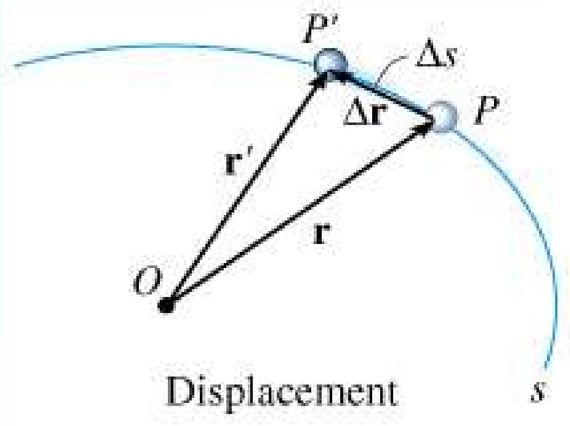
so the speed, v, is given by

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

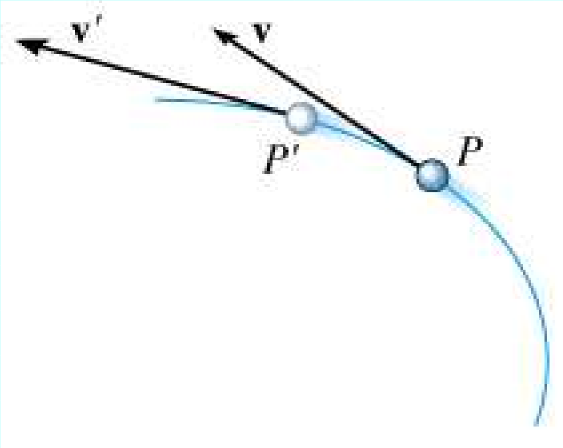
or

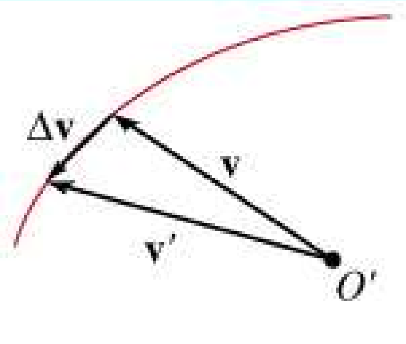
$$v = \frac{ds}{dt}$$





- Note that this means that we can determine the speed of the particle by differentiating the path function, $s(t)$, with respect to time





ACCELERATION

Average acceleration

- Suppose the particle has velocity **v** at time *t*, and velocity $\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$ at time $t + \Delta t$

- Then the average acceleration over the time interval is

$$\mathbf{a}_{\text{avg}} = \frac{\Delta\mathbf{v}}{\Delta t}$$

- We can study this time rate of change by translating the velocity vectors so that their tails coincide at some (arbitrary) fixed point *O'*

- Velocity arrowheads then touch points on a curve known as a **hodograph**, which is analogous to the particle path for the position vector

Instantaneous acceleration (acceleration)

- Again, we can take the limit $\Delta t \to 0$ in the expression for the average acceleration, and this gives the instantaneous acceleration, or simply the acceleration, **a**, of the particle

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

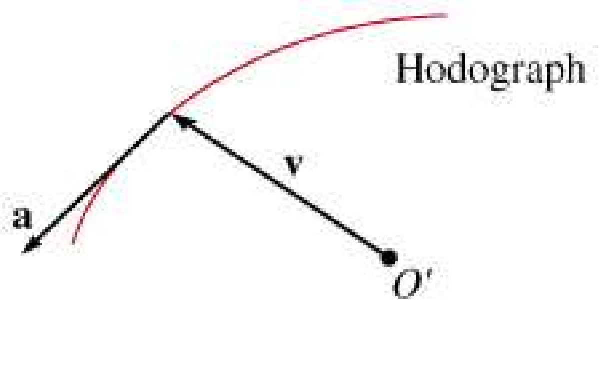
or

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

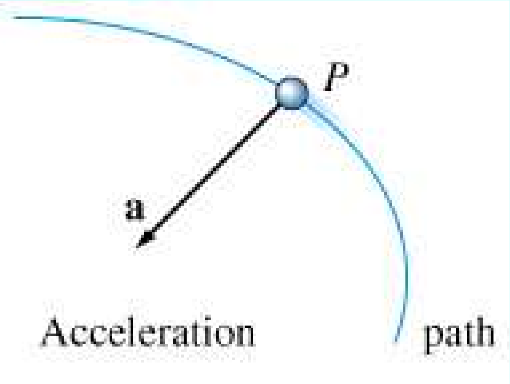
- Substituting $\mathbf{v} = d\mathbf{r} / dt$ in the above, we also have

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

(Instantaneous) Acceleration (continued)



- **NOTE:** The direction of **a** (and $d\mathbf{v}$) is always **tangent** to the **hodograph** and **not**, in general, tangent to the particle path



- In particular, note that **a** must account for the change in **direction** of **v** as well as the change in **magnitude** of **v**

- In order for particle to follow path, **a** must "swing" **v** towards "inside" ("concave side") of path as shown in the figure at the left