PHYS 170 Section 101
Lecture 11
September 28, 2018

Lecture Outline/Learning Goals

- Simplification of a force and couple system

- Sample problem involving replacement of force and couple system by equivalent force and couple moment

- Further simplification of a force and couple system
  - Concurrent force system
  - Coplanar force system
  - Parallel force system
  - Reduction to a wrench

Equivalence

- Central theme of lecture
- Consider force & (couple) moment systems which differ in detail (e.g. number of forces and/or couple moments, where forces and/or couple moments are applied) but which have identical physical effects
- Equivalence is often useful for simplifying descriptions of force/moment systems

Equivalence

4.7 Simplification of a Force and Couple System

- Force has potential to both translate and rotate a body, and the amount it does these depends on where and how the force is applied

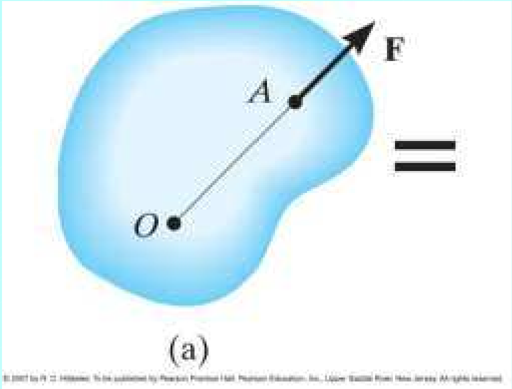
- Will often be interested in *simplifying* system of forces and couple moments acting on a body to a single resultant force and a single couple moment acting at some specified point *O*.

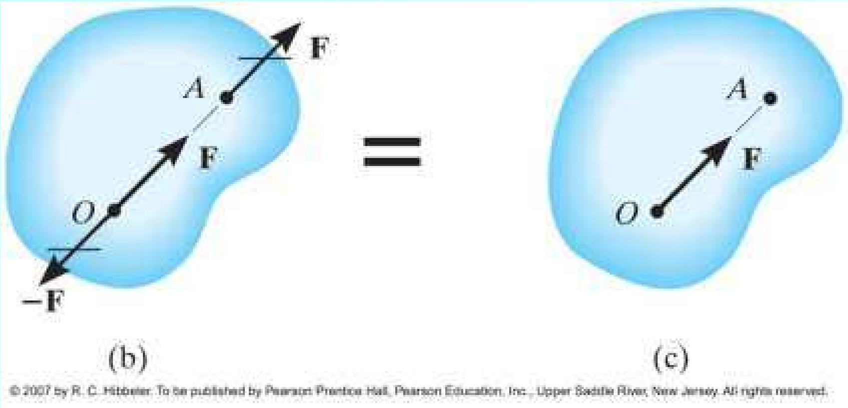
- In performing this simplification, will want to ensure that resultant force/couple moment system produces identical external effects on body as the original force/couple moment system: will then say that the systems are equivalent

- Will now discuss how to maintain such an equivalency for the case where a single force applied to a body at some point *A* is relocated to another point *O*

- Two subcases to consider
  - Point *O* is on the line of action of the force
  - Point *O* is not on the line of action of the force

Point *O* is On the Line of Action of the Force





- This case is straightforward
- As shown in the figure, can relocate force from *A* to *O* via intermediate step (Fig (b)), in which we introduce a copy of **F** and its negative **–F** at point *O*
- Force **F** at *A* cancels with force **–F** at *O*, and we are left with single force **F** at *O*. Cases (a) and (c) are thus equivalent (as is (b) for that matter)
- In general, then, can translate, or transmit, a force to any point in the body that lies along the line of action of the force, and an equivalent system will result

Relocation of force to point on line of action

(a) (b) (c)

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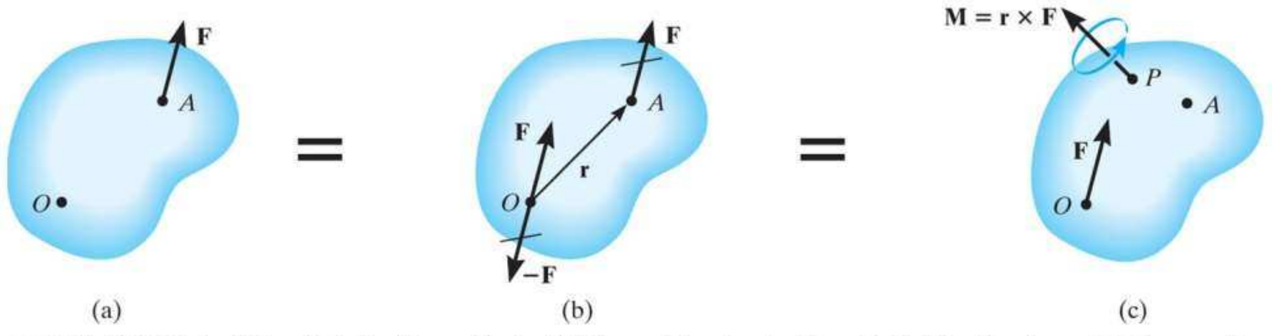
Point *O* is Not On the Line of Action of the Force

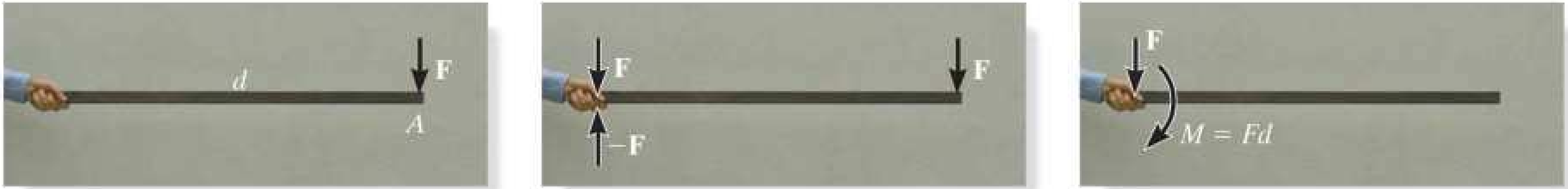


- This case is slightly more tricky
- Again proceed via intermediate step in which we introduce **F** and **–F** at point *O*
- Now note that **F** at *A* and –**F** at *O* form a couple moment, **M**, defined by

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Couple moment is a free vector, so can be located at *any point P* on the body
- Combined system of force relocated to *O* and couple moment located at arbitrary point, *P*, is equivalent to the original force applied at point *A*

Relocation of force to point *not* on line of action



(a) (b) (c)

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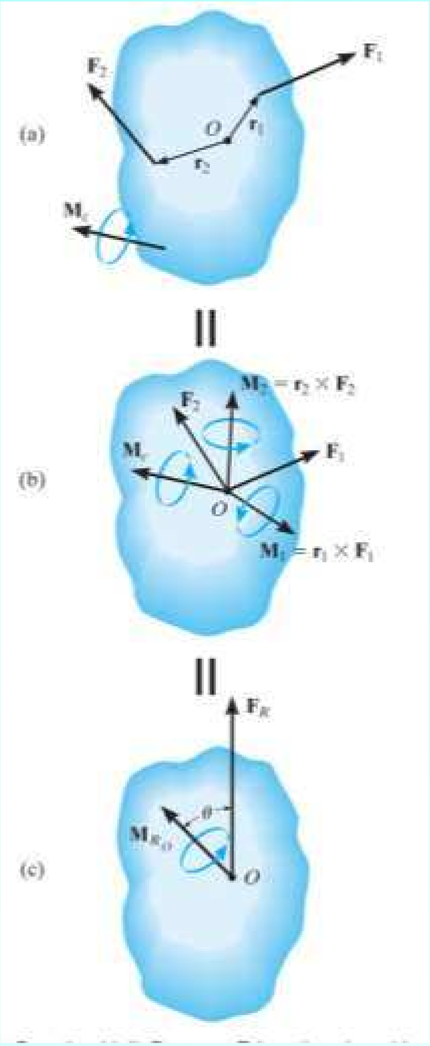
Resultants of a Force and Couple System



- Now consider body acted on by *system* of forces and couple moments
- To study external effects of system, often advantageous to replace system by equivalent single resultant force acting at some point, *O*, and a resultant couple moment
- Consider figure: Point *O* is *not* along line of action of either force, so in relocating forces to *O*, must apply couple moments

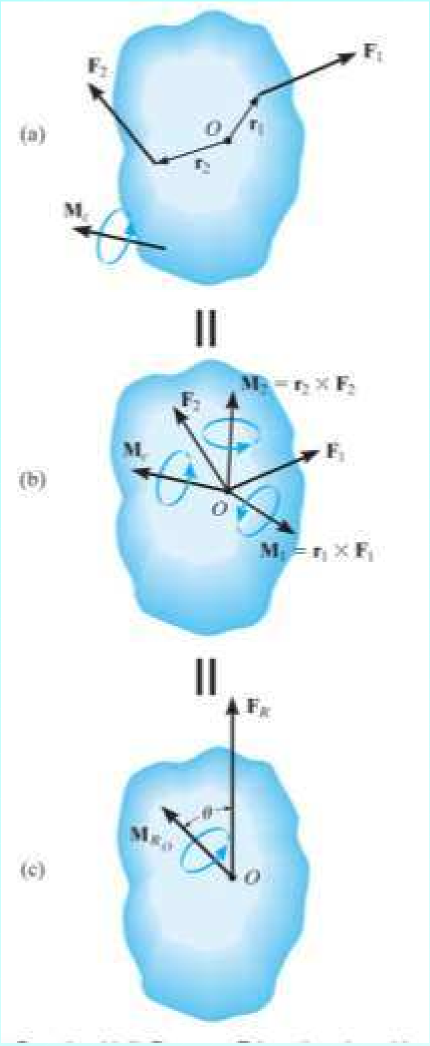
$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2$$

to maintain equivalence

- Thus have (at point *O*)

Equivalent resultant force: $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

Equivalent resultant couple moment: $(\mathbf{M}_R)_O = \mathbf{M} + \mathbf{M}_1 + \mathbf{M}_2$



- **NOTE:**
  - $\mathbf{F}_R$ is *independent* of location of *O* in body
  - $\mathbf{M}_1$, $\mathbf{M}_2$ and $(\mathbf{M}_R)_O$ are *not independent* of location of *O*
  - Nonetheless, $(\mathbf{M}_R)_O$ is a *free vector* so can be applied at any point (typically at *O*)
- Generalizing to case where arbitrary number of forces, couple moments act, we have (again, at point *O*)

Equivalent resultant force: $\mathbf{F}_R = \Sigma \mathbf{F}$

Equivalent resultant couple moment: $(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$

- **SPECIAL CASE:** If forces are all coplanar (say in the *xy* plane), and all couple moments are perpendicular to the plane (i.e. in the $\pm z$ directions), then have following 3 scalar equations

$$F_{R_x} = \Sigma F_x$$

$$F_{R_y} = \Sigma F_y$$

$$(M_R)_O = \Sigma M_O + \Sigma M$$

Problem 4-116 (page 169, 12th edition)

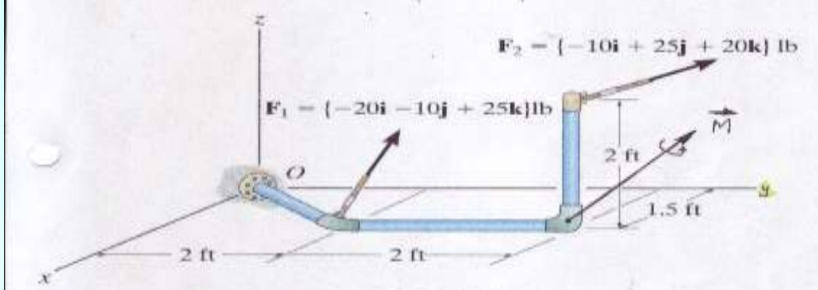
The pipe assembly is acted on by forces $\vec{F}_1$ and $\vec{F}_2$ and by a couple moment

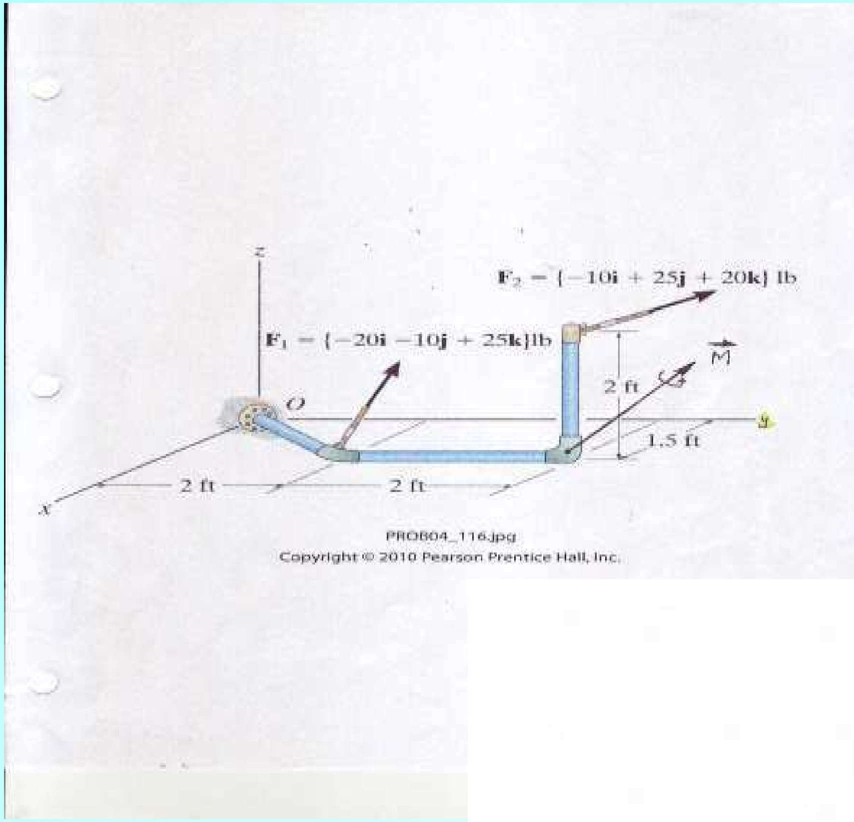
$\vec{M} = \left(5\vec{i} + 6\vec{j} + 7\vec{k}\right)$ lb·ft

(1) Determine the magnitude and coordinate direction angles of $\vec{M}$

(2) Determine the magnitude of each of the forces comprising the couple when the moment arm of the couple is 0.5 ft

(3) Replace the force-couple system by a resultant force and couple moment at O Express the results in Cartesian vector form





z
F2 = {−10i + 25j + 20k} lb
F1 = {−20i −10j + 25k}lb
M
O
2 ft
1.5 ft
y
x
2 ft
2 ft
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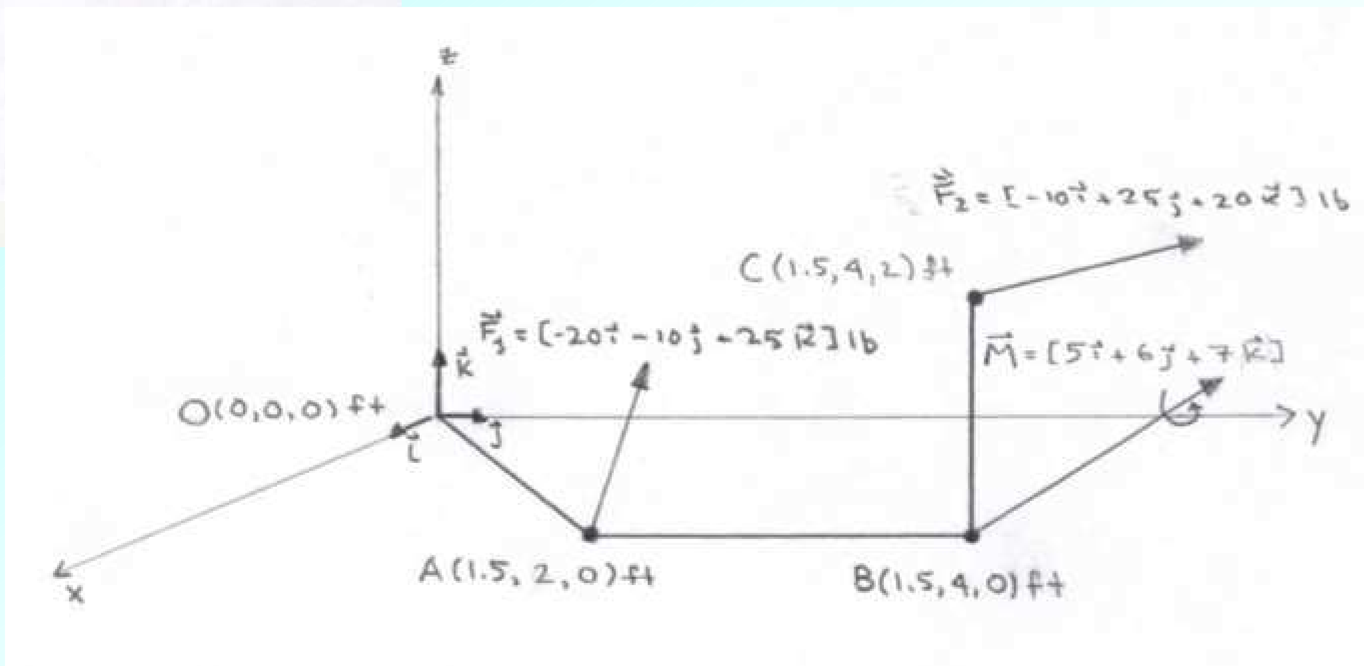
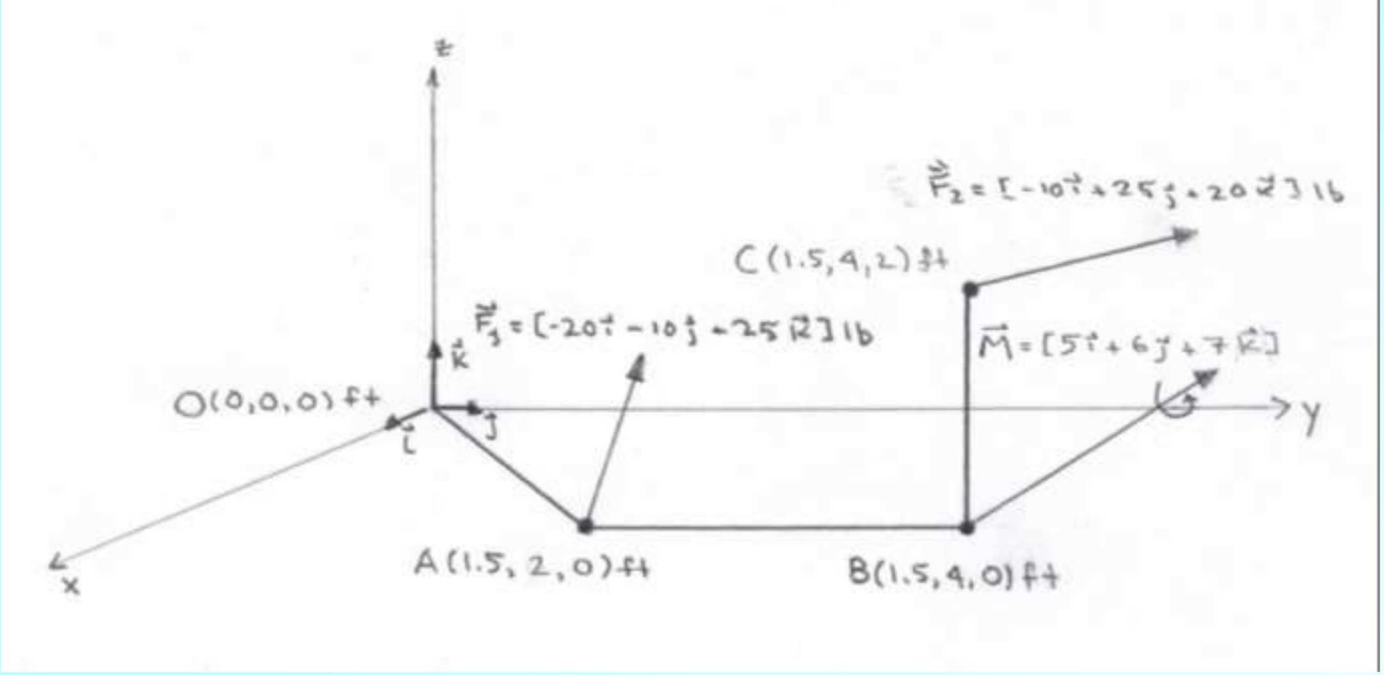


z
F2 = [−10i + 25j + 20k] lb
C(1.5, 4, 2) ft
F1 = [−20i − 10j + 25k] lb
M = [5i + 6j + 7k]
k
O(0,0,0) ft
i
j
y
x
A(1.5, 2, 0) ft
B(1.5, 4, 0) ft



Solution strategy

(1) Straightforward calculation of magnitude of $\vec{M}$ and coordinate direction angles from Cartesian form

(2) Use $M = Fd$, where M and d are both known

(3) Compute resultant force at O by summing forces; compute resultant couple moment at O using $(\vec{M}_R)_O = \sum \vec{M} + \sum (\vec{r} \times \vec{F})$

(1) Magnitude and cooordinate direction angles of $\vec{M}$

$$M = \sqrt{5^2 + 6^2 + 7^2}\ \text{lb} \cdot \text{ft} = 10.49\ \text{lb} \cdot \text{ft} = 10.5\ \text{lb} \cdot \text{ft}$$

$$\alpha = \cos^{-1}(5/10.49) = 61.5^\circ$$

$$\beta = \cos^{-1}(6/10.49) = 55.1^\circ$$

$$\gamma = \cos^{-1}(7/10.49) = 48.1^\circ$$

(2) Force magnitude when moment arm is 0.5 ft

$$F = M/d = 10.49/0.5\ \text{lb} = 21.0\ \text{lb}$$

(3) Resultant force

$$\vec{F}_R = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 = \left[-20\vec{i} - 10\vec{j} + 25\vec{k}\right]\ \text{lb} + \left[-10\vec{i} + 25\vec{j} + 20\vec{k}\right]\ \text{lb}$$

$$= \left[-30.0\vec{i} + 15.0\vec{j} + 45.0\vec{k}\right]\ \text{lb}$$

(3 cont.) Resultant couple moment $\left(\vec{M}_R\right)_O$ at O (suppressing units)

Coordinates

$A(1.5,\ 2,\ 0)$

$C(1.5,\ 4,\ 2)$

$O(0,\ 0,\ 0)$

Position vectors

$$\vec{r}_{OA} = 1.5\vec{i} + 2\vec{j}$$

$$\vec{r}_{OC} = 1.5\vec{i} + 4\vec{j} + 2\vec{k}$$

(3 cont.) Resultant couple moment $\left(\vec{M}_R\right)_O$ at O (suppressing units)

$$\left(\vec{M}_R\right)_O = \sum \vec{M} + \sum\left(\vec{r}\times\vec{F}\right) = \sum \vec{M} + \vec{r}_{OA}\times\vec{F}_1 + \vec{r}_{OC}\times\vec{F}_2$$

$$= 5\vec{i} + 6\vec{j} + 7\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix}$$

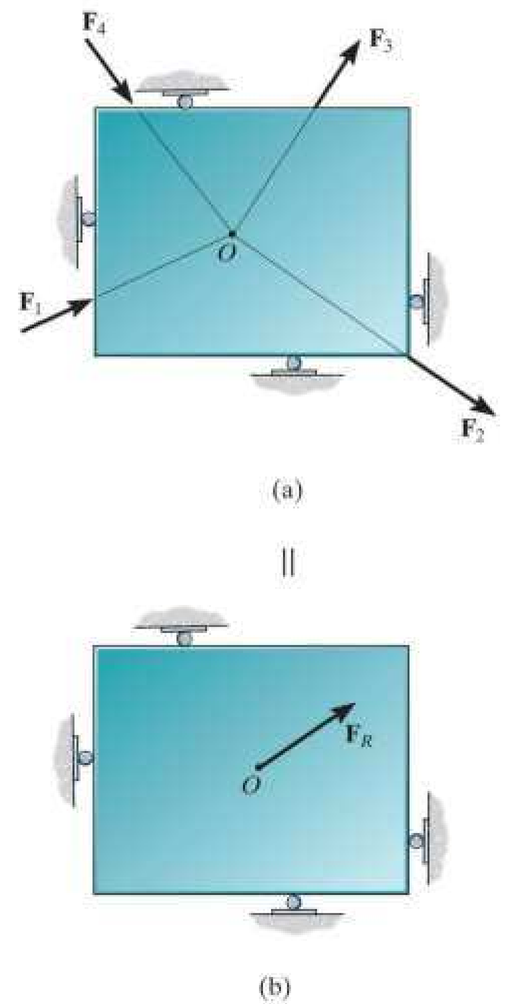
$$= 5\vec{i} + 6\vec{j} + 7\vec{k} + (50-0)\vec{i} - (37.5-0)\vec{j} + (-15+40)\vec{k} +$$

$$(80-50)\vec{i} - (30+20)\vec{j} + (37.5+40)\vec{k}$$

$$= (85.0\vec{i} - 81.5\vec{j} + 110\vec{k}\ )\,\mathrm{lb}\cdot\mathrm{ft}$$

4.8 Further Simplification of a Force and Couple System

Concurrent Force System



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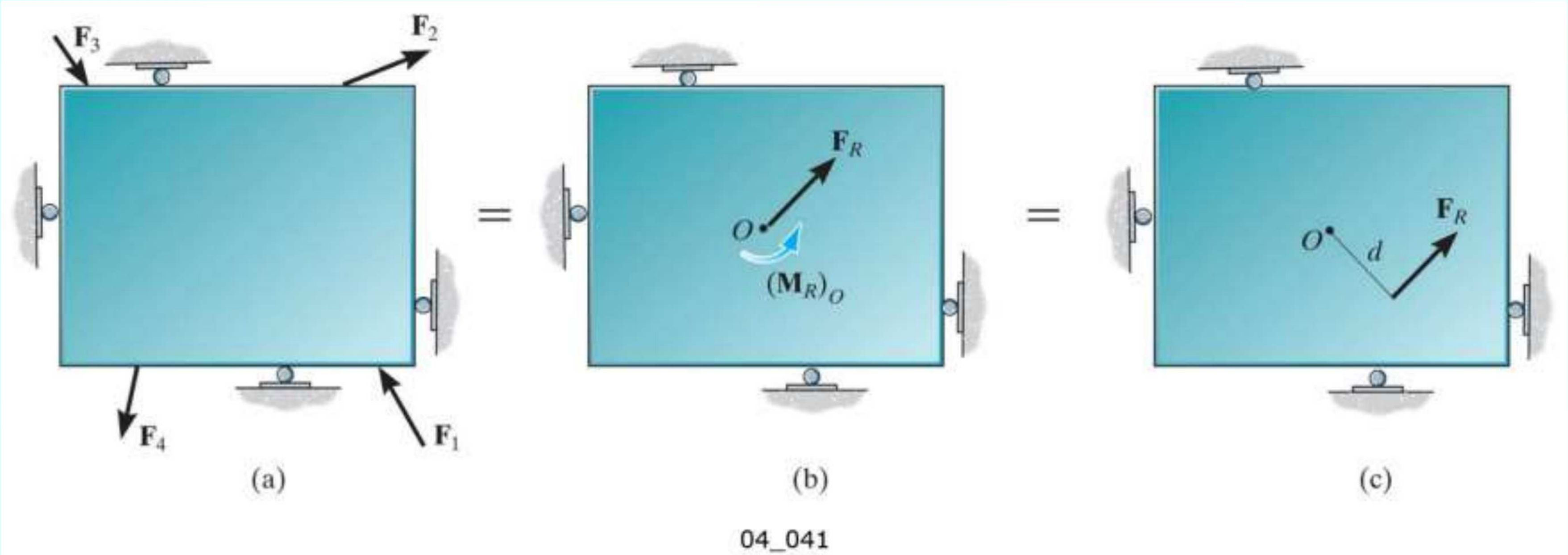
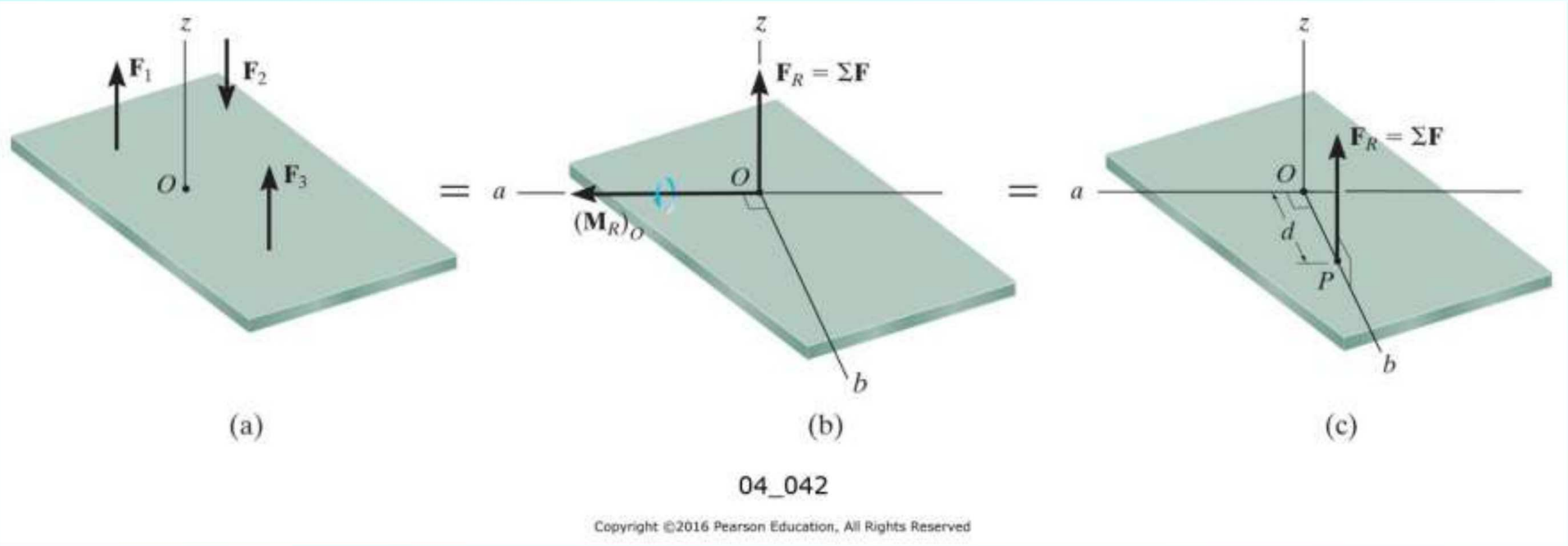
In a concurrent force system, all lines of action of the forces intersect at some point *O*. Thus there are no moments produced about this point, and the system can be equivalently represented by the single resultant force $F_R = \sum F$ acting at *O*.

Coplanar Force System



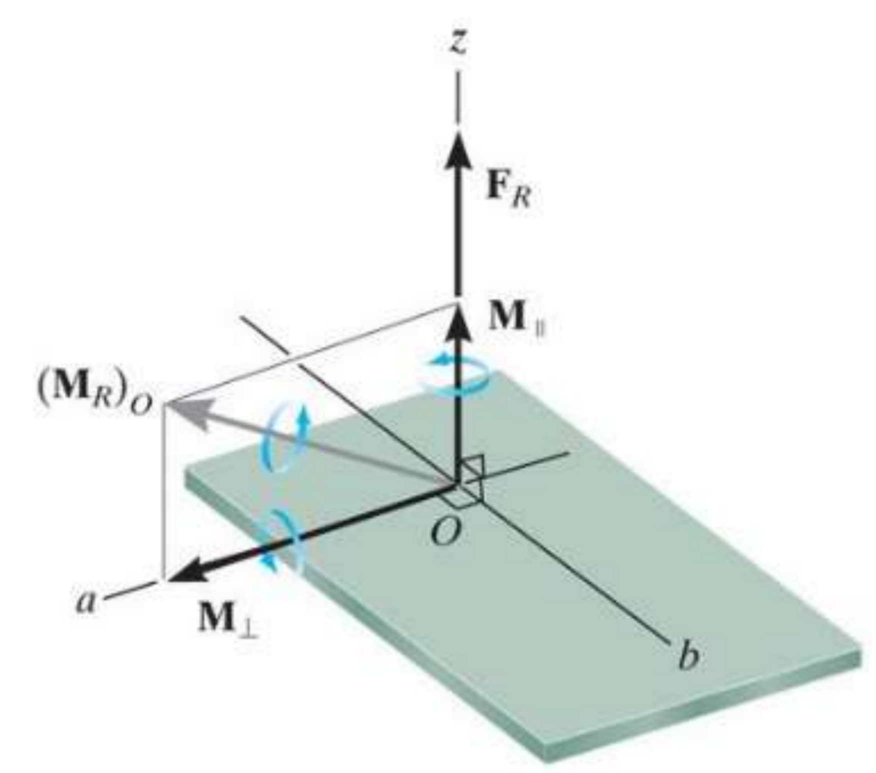
All the lines of action of the $\vec{F}_i$ lie in a single plane so resultant force $\vec{F}_R = \sum \vec{F}$ lies in the plane as well. Also, all moments about O are perpendicular to the plane so the resultant moment $(\vec{M}_R)_O$ and $\vec{F}_R$ are perpendicular. Therefore, $(\vec{M}_R)_O$ can be replaced by moving $\vec{F}_R$ a perpendicular distance $d = (M_R)_O / F_R$ away from O so that $\vec{F}_R$ generates the same moment about O.

Parallel Force System



In the example shown here all forces are parallel to the z axis so the resultant force $\vec{F}_R = \sum \vec{F}$ at point O is also parallel to it. Moment due to each force lies in the plane of the plate so the resultant couple moment $(\vec{M}_R)_O$ also lies in that plane, along some moment axis a ($\vec{F}_R$ and $(\vec{M}_R)_O$ are perpendicular). The system can be further reduced by moving $\vec{F}_R$ a distance $d = (M_R)_O / F_R$ along an axis b that is perpendicular to a to a point P, as shown in Fig. (c), so that the resulting moment of $\vec{F}_R$ about O is $(\vec{M}_R)_O$.

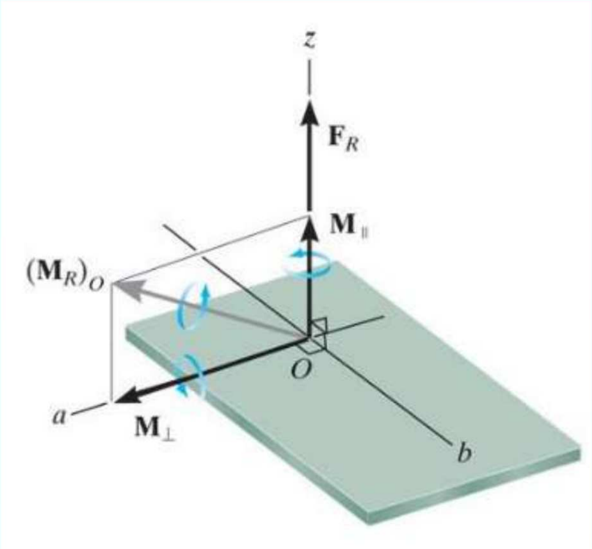
Reduction to a Wrench



(a)

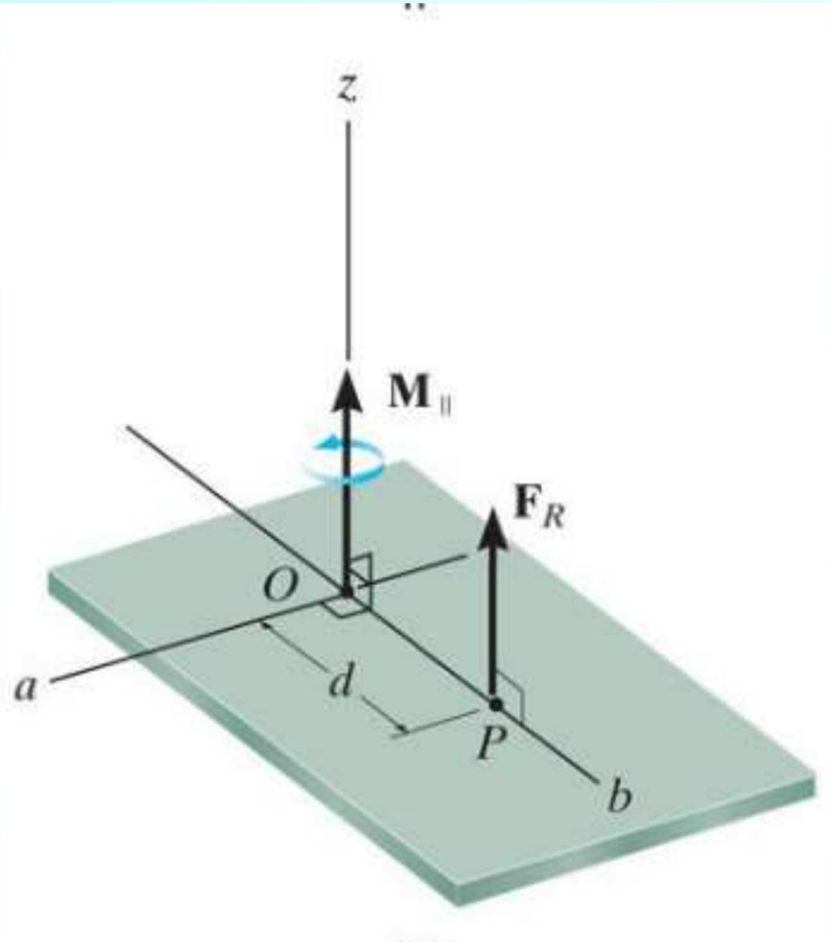
In general, as is the case in the figure, $\vec{F}_R$ and $(\vec{M}_R)_O$ will not be perpendicular. However, $(\vec{M}_R)_O$ can be resolved into components $\vec{M}_\parallel$ and $\vec{M}_\perp$ which are parallel and perpendicular, respectively, to $\vec{F}_R$.

Reduction to a Wrench



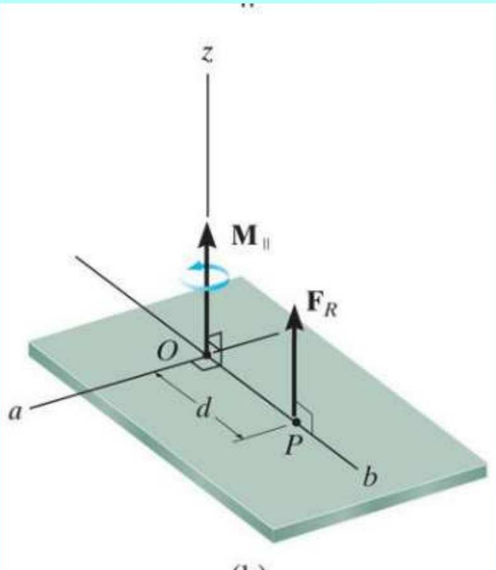
(a)

$\vec{M}_\perp$ can be eliminated as in the previous construction by moving $\vec{F}_R$ a distance $d = (M_R)_O / F_R$ along an axis b to a point P as in Fig. (b).



(b)

Reduction to a Wrench

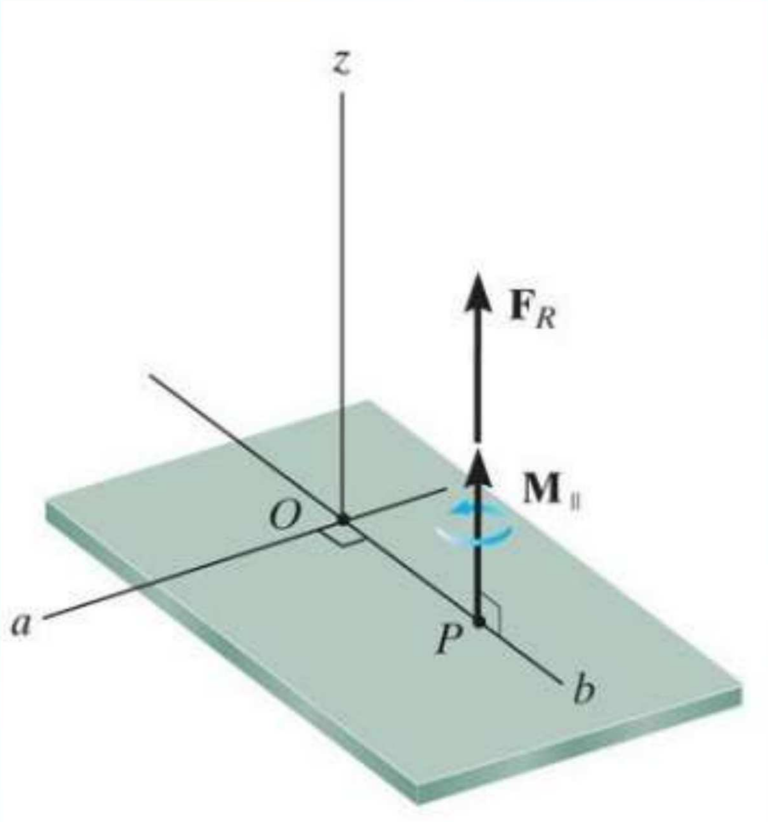


(b)

Then, since $\vec{M}_{\parallel}$ is a free vector,

it can be relocated to point P as in Fig. (c).

The combination of a resultant force $\vec{F}_R$ and a collinear (parallel or anti-parallel) couple moment $\vec{M}_{\parallel}$ is called a wrench or screw and is the simplest equivalent of a general force and couple moment system.



(c)

Colinear (parallel) vectors

Two vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

are colinear (parallel) if and only if there is some constant $c \neq 0$ such that

$$\vec{B} = c\vec{A}$$

so that

$$\vec{B} = cA_x\vec{i} + cA_y\vec{j} + cA_z\vec{k}$$