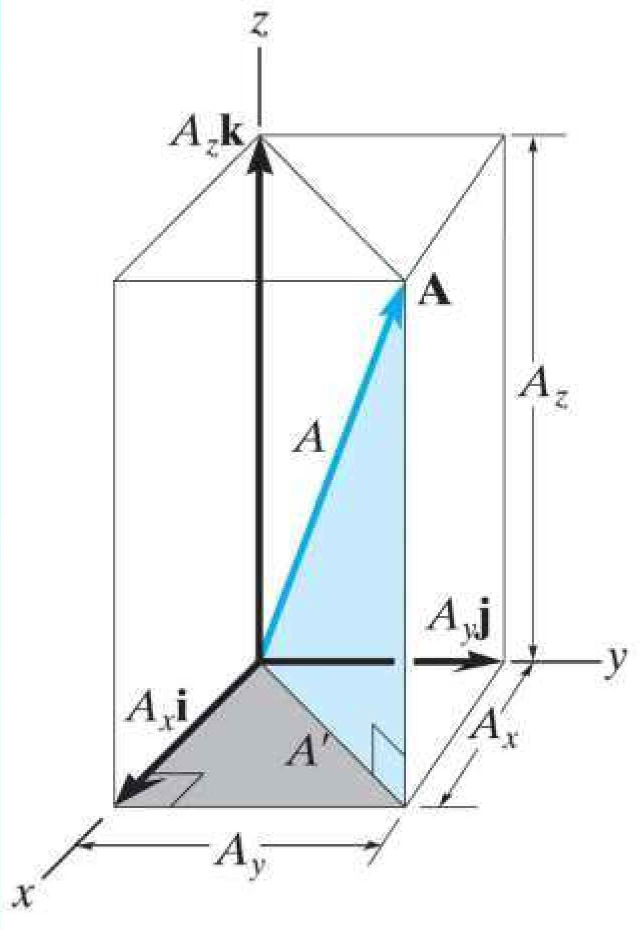
PHYS 170 Section 101
Lecture 4
September 12, 2018

Lecture Outline/Learning Goals

- Finish introduction to Cartesian vectors
  - Cartesian vector: direction, coordinate direction angles, direction cosines
  - Operations with Cartesian vectors
- Concurrent force systems
- Interpretation of 3D figures
- Sample problem (concurrent force system)

CARTESIAN VECTOR REPRESENTATION & MAGNITUDE OF A CARTESIAN VECTOR



$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

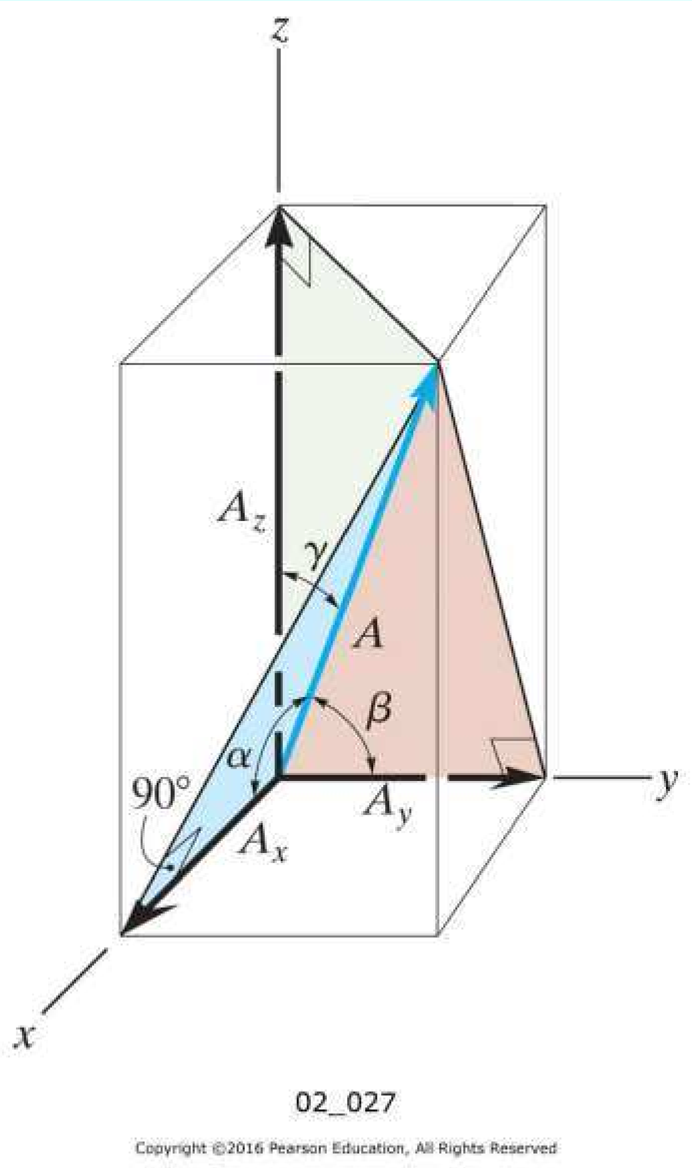
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

A_x, A_y and A_z can have either sign in general.

Rene Descartes (1596-1650)

One of Descartes' most enduring legacies was his development of Cartesian or analytic geometry, which uses algebra to describe geometry. He "invented the convention of representing unknowns in equations by *x*, *y*, and *z*, and knowns by *a*, *b*, and *c*." … (Wikipedia)

DIRECTION OF A CARTESIAN VECTOR

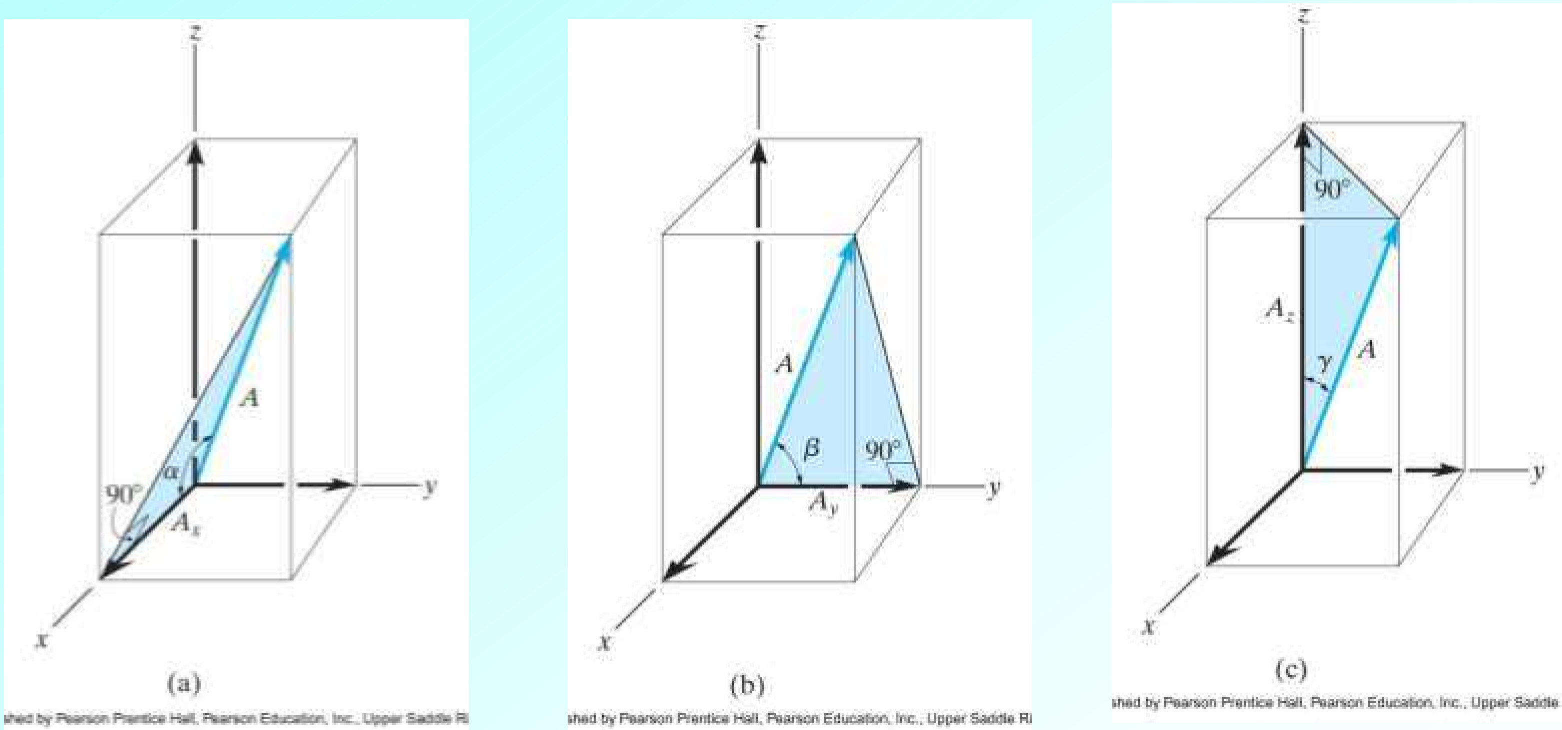


- Introduce coordinate direction angles

$$\alpha, \ \beta, \ \gamma$$

as measured between the tail of the vector and the positive x, y, z axes located at the tail

DIRECTION COSINES



$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

- Consider unit vector in direction of $\mathbf{A}$

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

- Therefore have

$$\mathbf{u}_A = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

- Important relationship (since $\mathbf{u}_A$ is a unit vector)

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

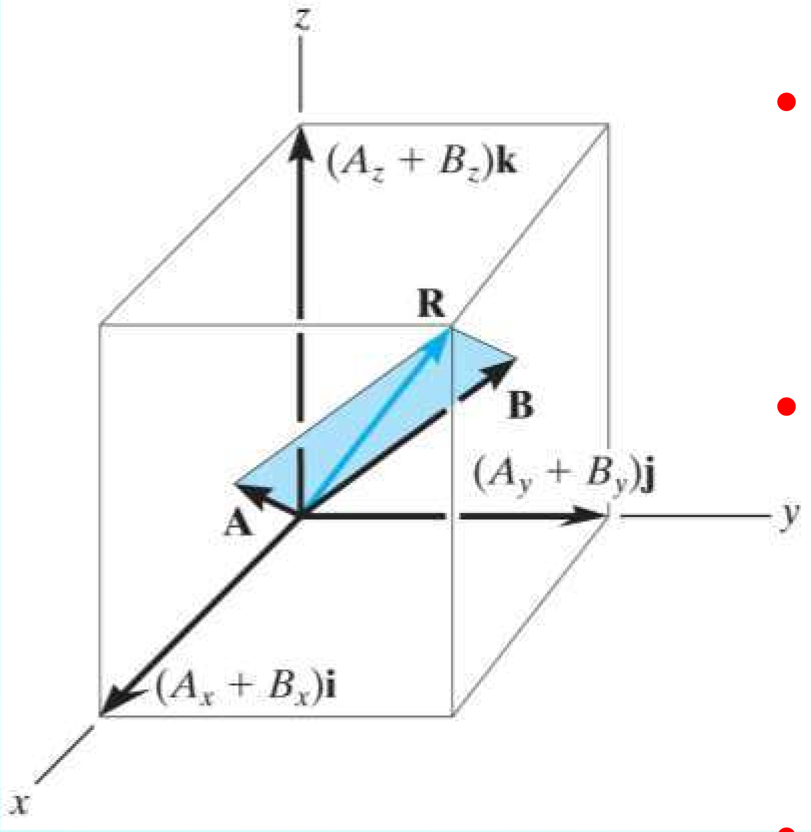
RELATION TO CARTESIAN VECTOR FORM

- Will sometimes be given a vector in terms of its magnitude and direction angles

- Can then convert to Cartesian vector form using

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\mathbf{i} + A\cos\beta\mathbf{j} + A\cos\gamma\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}$$

ADDITION & SUBTRACTION OF CARTESIAN VECTORS



- EASY!! Simply add/subtract corresponding components

- ADDITION

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$
$$= (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$$

- SUBTRACTION

$$\mathbf{R}' = \mathbf{A} - \mathbf{B}$$
$$= (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k}$$

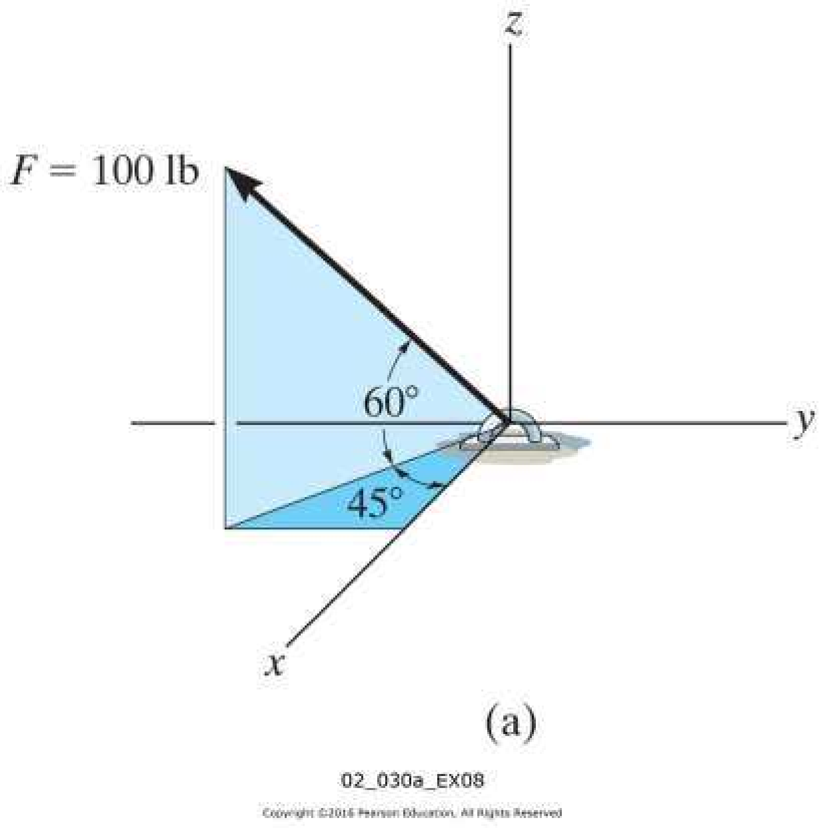
CONCURRENT FORCE SYSTEMS

- Can extend this technique of vector addition to system of an arbitrary number of concurrent (simultaneously applied) forces

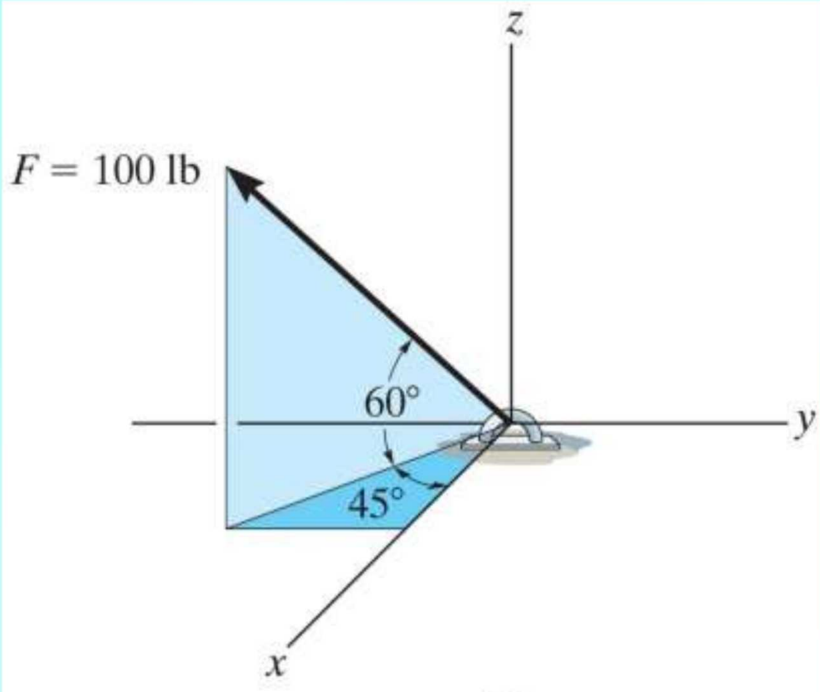
- General resultant force is given by

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Interpreting Three-dimensional (3D) Figures (expressing vectors in Cartesian form)



Express **F** as a Cartesian vector



(a)

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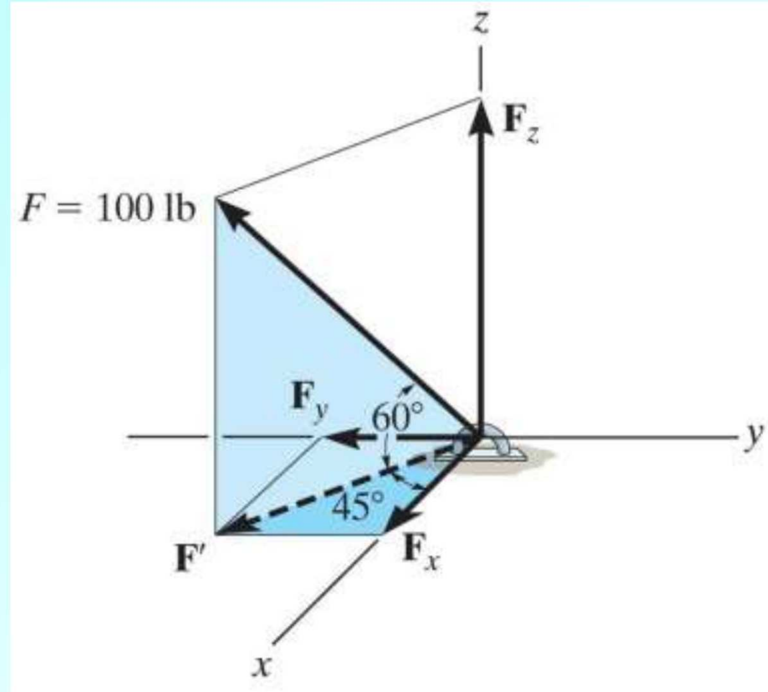


(b)

02_030b_EX08

$$|\vec{F}'| = |\vec{F}|\cos 60^\circ = (100\text{ lb})\cos 60^\circ = 50\text{ lb}$$

$$F_x = |\vec{F}'|\cos 45^\circ = (50\text{ lb})\cos 45^\circ = 35.4\text{ lb}$$

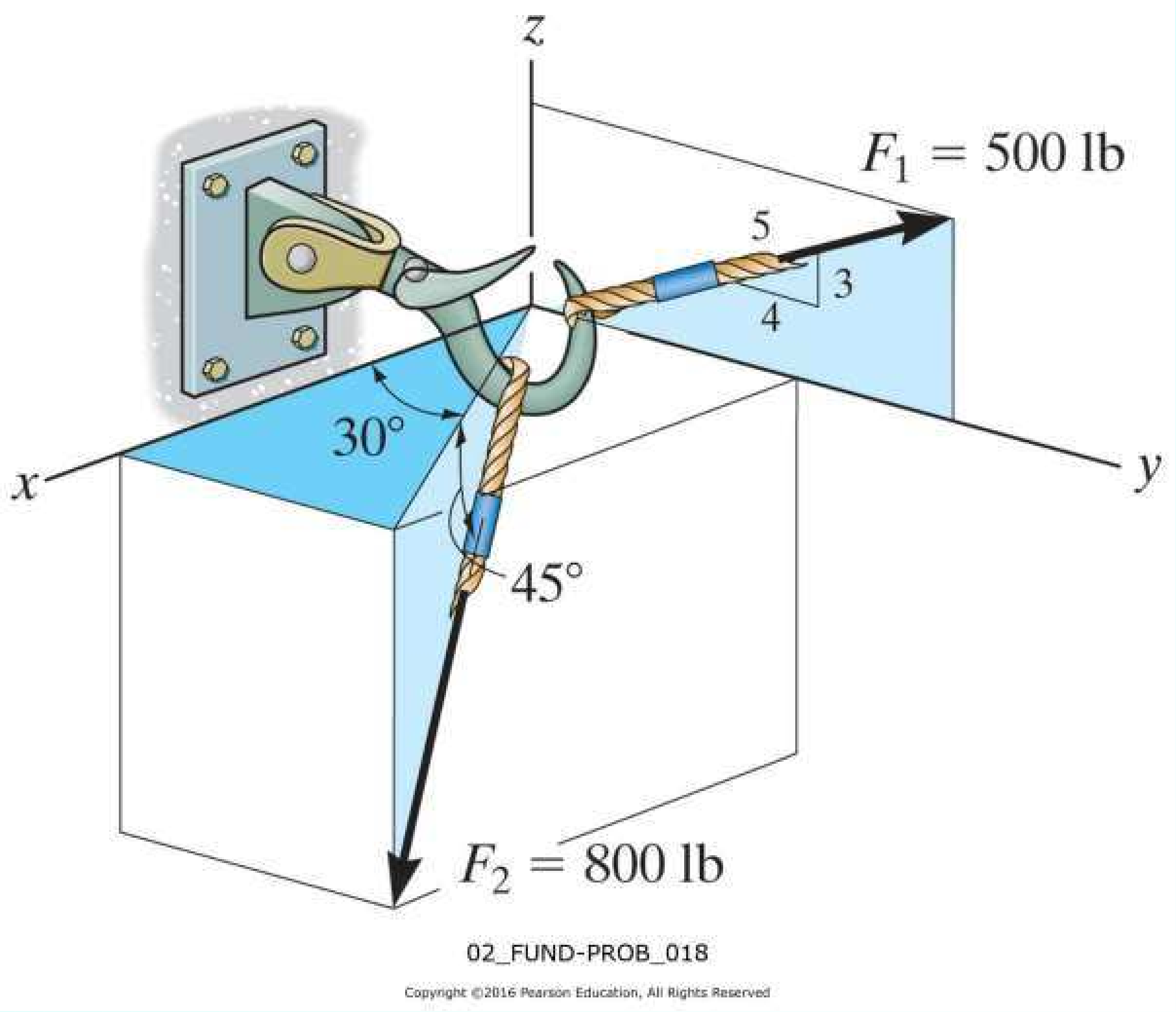
$$F_y = -|\vec{F}'|\sin 45^\circ = -(50\text{ lb})\sin 45^\circ = -35.4\text{ lb}$$

$$F_z = |\vec{F}|\sin 60^\circ = (100\text{ lb})\sin 60^\circ = 86.6\text{ lb}$$

$$\vec{F} = (35.4\,\vec{i} - 35.4\,\vec{j} + 86.6\,\vec{k})\text{ lb}$$

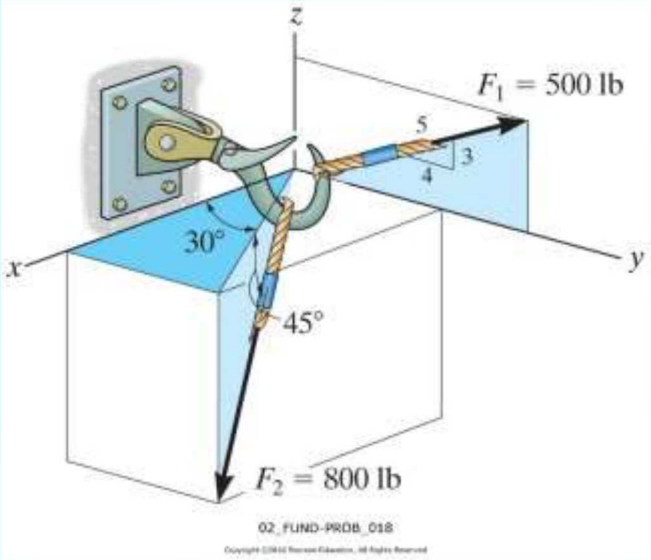
IMPORTANT: F_x, F_y and F_z are the components of the force $\vec{F}$, not the magnitudes of the vectors $\vec{F}_x, \vec{F}_y$ and $\vec{F}_z$ (text differs)

Problem F2-18 (page 51)



Determine the resultant force acting on the block

Shaded blue triangles always have one vertex that concides with origin. Among other things means that vector $\vec{F}_1$ lies in the yz plane (i.e. no x component). Resolution of $\vec{F}_2$ into components follows same two-step procedure as previously to determine F_{2x} and F_{2y}



$$F_{1y} = \left(\frac{4}{5}\right)(500) = 400 \text{ lb}$$

$$F_{1z} = \left(\frac{3}{5}\right)(500) = 300 \text{ lb}$$

$$F_{2x} = (800)(\cos 45^\circ)(\cos 30^\circ) = 489.90 \text{ lb}$$

$$F_{2y} = (800)(\cos 45^\circ)(\sin 30^\circ) = 282.84 \text{ lb}$$

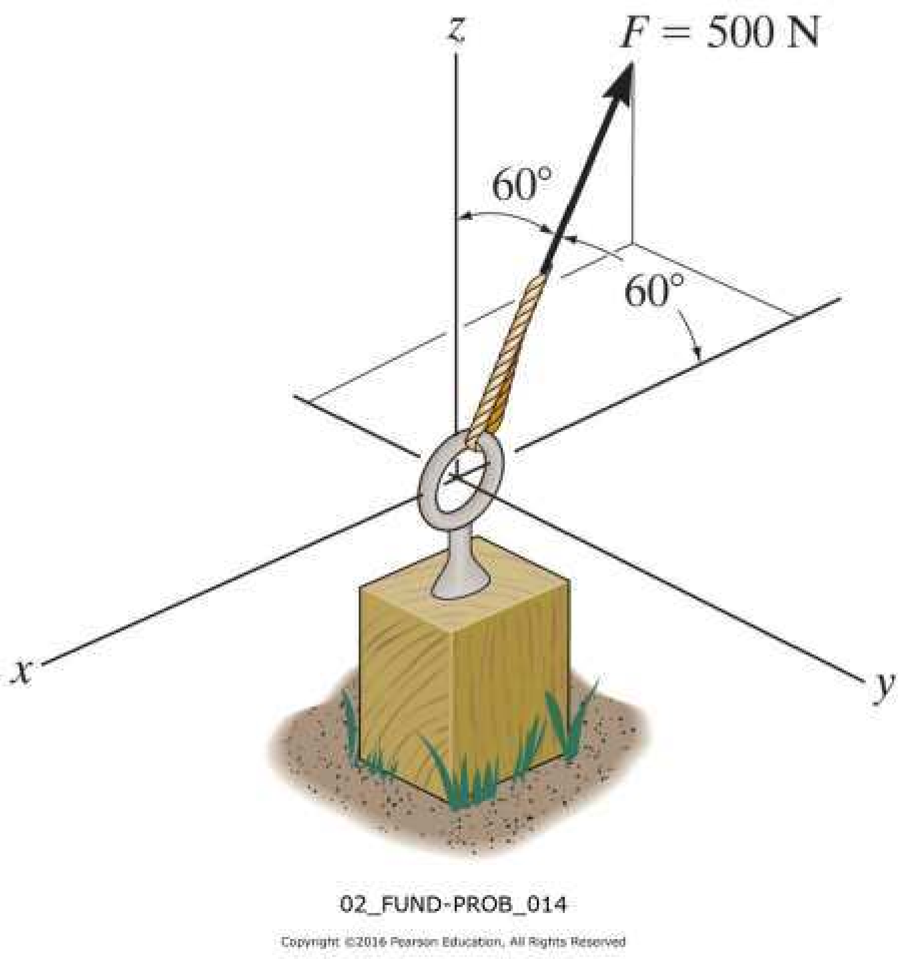
$$F_{2z} = -(800)(\sin 45^\circ) = -565.69 \text{ lb}$$

$$\vec{F}_R = \sum F_x \, \vec{i} + \sum F_y \, \vec{j} + \sum F_z \, \vec{k}$$

$$= (490 \, \vec{i} + 683 \, \vec{j} - 266 \, \vec{k}) \text{ lb}$$

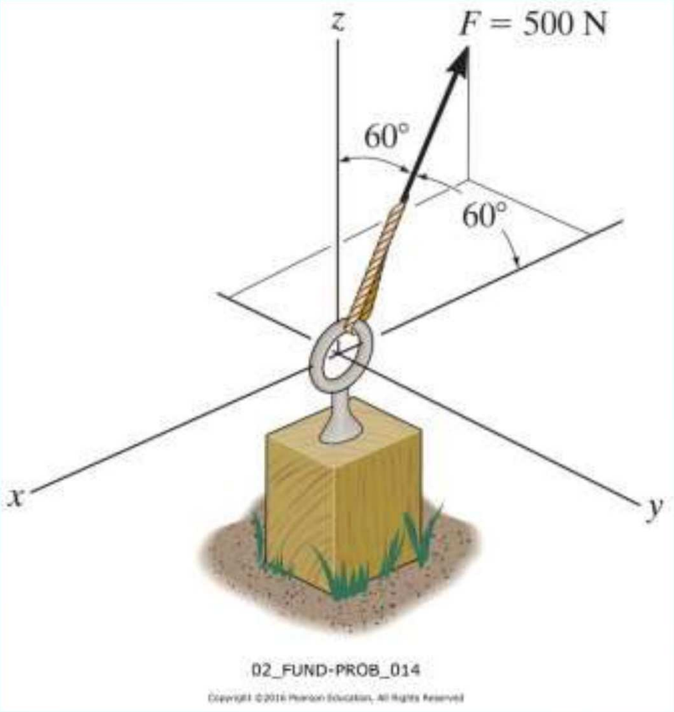
Note that we have suppressed units in the intermediate steps of the calculation. This is perfectly acceptable practice. Just ensure that your final answers ALWAYS include units.

Problem F2-14 (page 51)



Express the force as a Cartesian vector

In this example, the angles which are given *are* coordinate direction angles. Specifically, we are given $\alpha = 60^\circ$ and $\gamma = 60^\circ$. From those, and the equation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ we can determine β.



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos\beta = \sqrt{1 - \cos^2 60^\circ - \cos^2 60^\circ} = \pm 0.70711$$

We choose $\cos\beta = -0.70711$ since $F_y < 0$ from the figure.

Expressing the force as a Cartesian vector we have

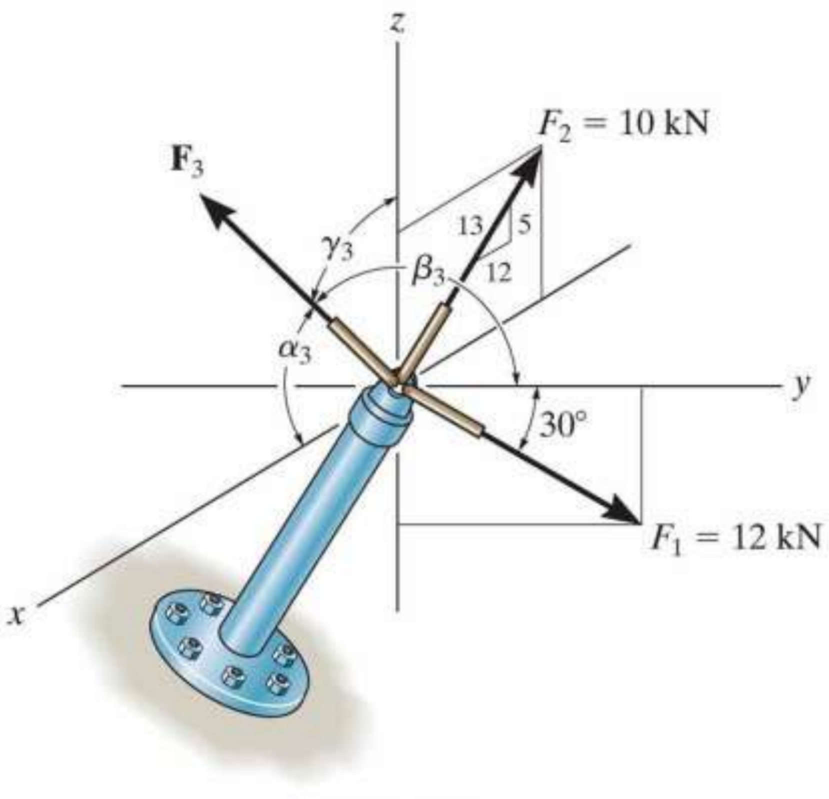
$$F = F\,\vec{u}_F$$

$$= (500)(-\cos 60^\circ\,\vec{i} - 0.70711\,\vec{j} + \cos 60^\circ\,\vec{k})$$

$$= (-250\,\vec{i} - 354\,\vec{j} + 250\,\vec{k})\text{ N}$$

Problem 2-79, (page 54, 12th edition)

Specify the magnitude of $\vec{F}_3$ and its coordinate direction angles α_3, β_3 and γ_3 so that the resultant force is $9\,\vec{j}$ kN



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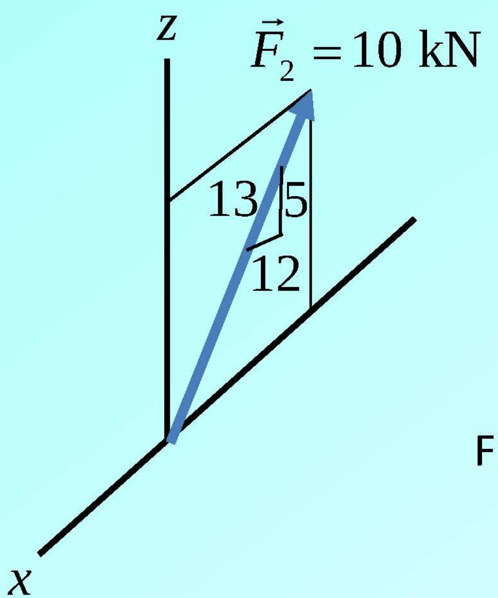


Fig. (a)

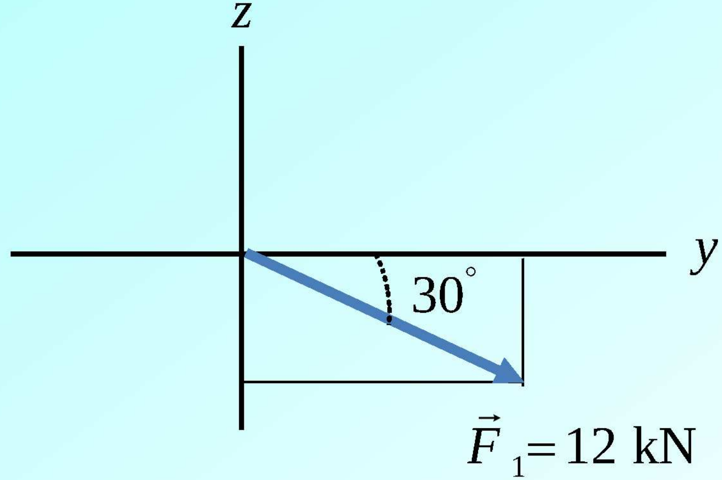


Fig. (b)

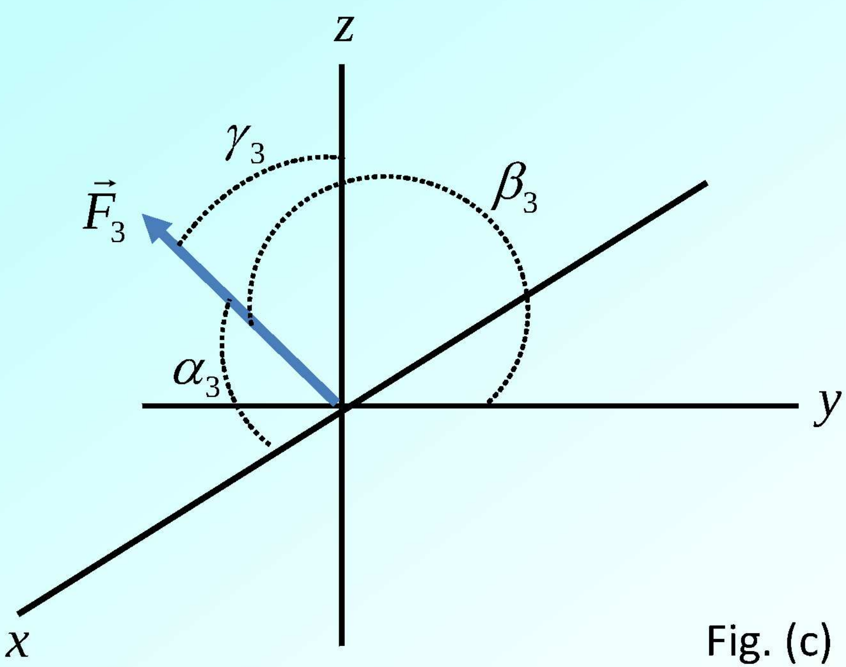
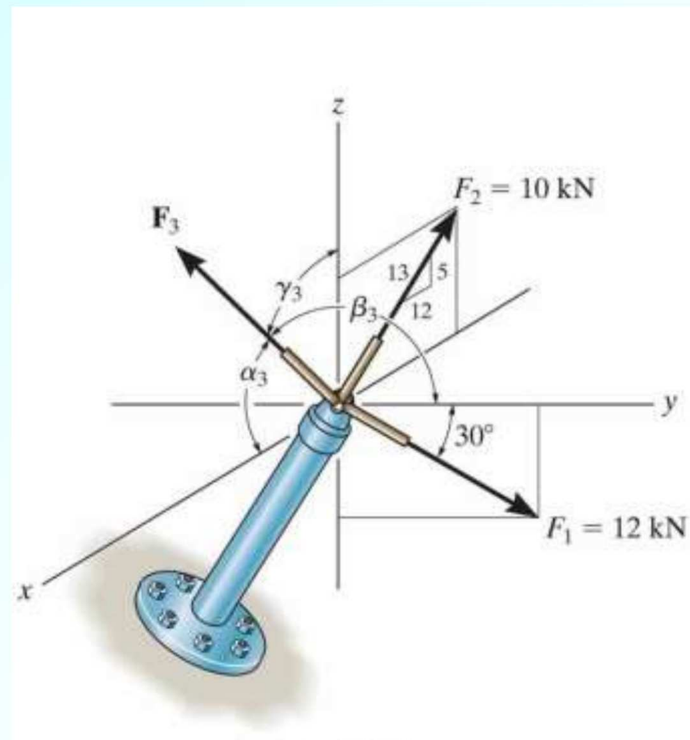


Fig. (c)



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